AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative Masters Programme in Economics for Anglophone Africa (Except Nigeria)

JOINT FACILITY FOR ELECTIVES (JFE) 2012

JUNE – SEPTEMBER

ECONOMETRICS THEORY AND PRACTICE I

First Semester: Final Examination

Duration: 3 Hours **Date: Friday, August 3, 2012**

INSTRUCTIONS:

1. Answer **FOUR (4)** questions: **TWO** questions in **Section A** and **TWO** in **Section B**.
2. All questions carry equal marks.
3. Present your work in a clear and orderly manner.

Section A:

Answer TWO Questions from this Section

Question 1

Consider the following model

$$y_i = b_1 + b_2 x_{i2} + b_3 x_{i3} + u_i \qquad i = 1, 2, ..., N$$

Where (y_i, x_{i2}, x_{i3}) are observed and have finite moments, and u_i is an unobserved error term. Suppose this model is estimated by ordinary least squares. Denote the OLS estimator by $\hat{\beta}$.

(i) What are the essential conditions required for unbiasedness of $\hat{\beta}$? What are the essential conditions required for consistency of $\hat{\beta}$? Explain the difference between unbiasedness and consistency. **(4 points)**

(ii) Suppose that $\text{cov}(u_i, x_{i3}) \neq 0$ Give two examples where one can expect a non-zero correlation between a regressor, x_{i3}, and the error term, u_i. **(3 points)**

(iii) In this case, is it appropriate to use the OLS estimator? **(3 points)**

(iv) Explain how an instrumental variable, let us call it, z_i, leads to a new moment condition and, consequently an alternative estimator for β. **(5 points)**

Question 2

Consider the linear regression model

$$y = X\beta + \epsilon$$

where each component of X is viewed as being an *exogenous regressor* if it is uncorrelated with the error term, ϵ, or *an endogenous regressor* if it is correlated with the error term. Suppose, there are components of X which are endogenous, and that there is a vector of instruments Z that satisfies $E[\epsilon_i|z_i] = 0$; and also suppose that the number of instruments, l, exceeds the number of regressors, q. That is the model is over-identified, $(l > p)$.

(i) Explain why the generalized method of moments (GMM) estimator would be the appropriate estimator in this case. **(6 points)**

(ii) If we let W_n with the property $p\lim W_N = W$, to be an $l \times l$ full-rank symmetric weighting matrix, the GMM estimator in the linear instrumental variable model is given by $\hat{\beta}_{GMM} = [X'ZW_NZ'X]^{-1}X'ZW_NZ'y$. Show that $plim\hat{\beta}_{GMM} = \beta$. **(6 points)**

(iii) To what extent is the claim that *"the method of moments, ordinary least squares, and instrumental variable estimators are all special cases of the GMM estimator"* true? **(3 points)**

Question 3

Given the AR(1) stochastic process

$$(1 - \beta L)y_t = c + \varepsilon_t$$

where ε_t is a white noise process, $\varepsilon_t \sim N(0, \sigma^2)$.

(i) Under which conditions is the process stationary? **(2 points)**

(ii) Derive its moving average representation. **(5 points)**

(iii) Find its mean, variance; and first, and second autocovariances (γ_1, and γ_2), of the process, if $\beta = -0.4$, $c = 20$, and $\sigma^2 = 1.2$. **(8 points)**

Section B:

Answer TWO Questions from this Section

Question 4

Two series of 200 observations were generated by the following equations:

$$X_t = X_{t-1} + u_t, \qquad u_t \sim IIN(0,1) \text{.....(1)}$$

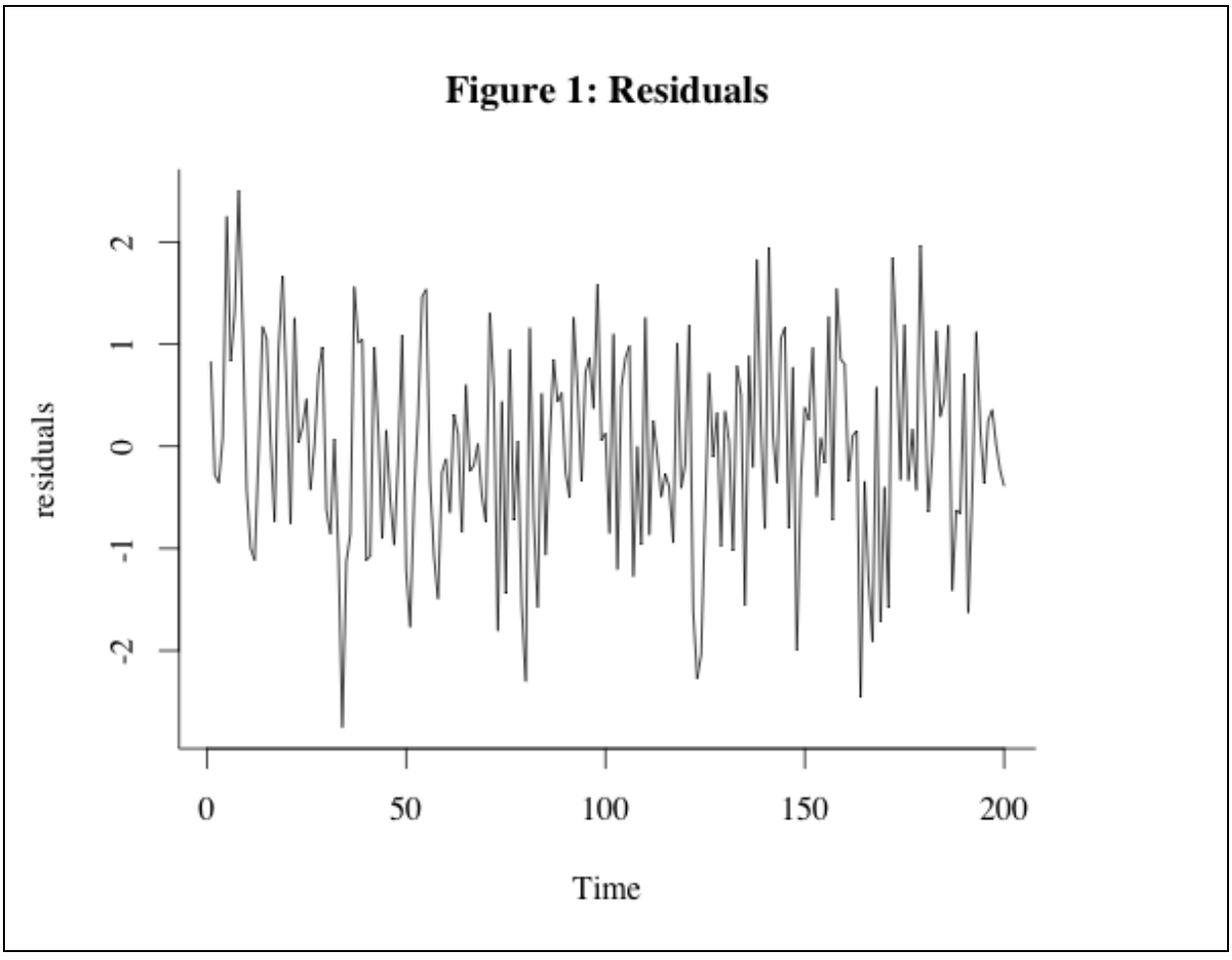
$$Y_t = \alpha + \beta X_t + \varepsilon_t \ldots . \varepsilon_t \sim IIN(0,1) \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

where u_t and ε_t are mutually independent innovations.

The relationship in equation (2) is estimated by OLS using the 200 observations of time series. The results are given in Table 1 below. Residuals are plotted in Figure 1 below.

Table 1: Regression of Y_t on X_t: OLS results

Variable	Coefficient	Standard error	t-ratio
Intercept	-0.039	0.129	-0.303
X_t	0.753	0.013	57.450
T=200		R^2= 0.93	DW = 1.98



(i) Are there indications that the relationship between the two variables is spurious? **(4 points)**

(ii) Explain what we mean by '*spurious regressions'*. **(2 points)**

(iii) Are there indications that there is a cointegrating relationship between Y_t and X_t? If the variables are cointegrated, what is the cointegrating vector? **(4 points)**

(iv) Explain what '*cointegrating relationship'* means. **(2 points)**

(v) Briefly, explain what we mean by a '*superconsistent estimator*'? **(3 points)**

Question 5

The following equation is fundamental in testing for cointegration, using the Johansen procedure:

$$\mathbf{DY_t} = \sum_{i=1}^{p-1} \boldsymbol{\pi}_i \mathbf{DY_{t-i}} + \boldsymbol{\pi} \mathbf{Y_{t-p}} + u_t \quad .....(*)$$

Equation (*) which is the Vector error correction model (VECM) is essentially a re-parameterized vector autoregressive model (VAR), derived from the following $VAR(p)$ model:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + A_3 Y_{t-3} + \text{----} + A_p Y_{t-p} + u_t$$

(a) Using the concept of a rank of a matrix, explain how you would use matrix $\boldsymbol{\pi}$ to determine if the variables in the system are cointegrated, and the number of cointegrating vectors. **(10 points)**

(b) In what sense is the cointegration test a unit root test in the multivariate time series framework? **(5 points)**

Question 6

(a) Briefly, explain the Sims' (1980) modeling philosophy. **(3 points)**

Given a simple vector autoregressive model:

$$y_t = \alpha_{10} + \beta_{11} y_{t-1} + \text{----} + \beta_{1p} y_{t-p} + \gamma_{11} x_{t-1} + \text{----} + \gamma_{1p} x_{t-p} + u_{1t}$$

$$x_t = a_{20} + b_{21} y_{t-1} + \text{----} + b_{2p} y_{t-p} + g_{21} x_{t-1} + \text{----} + g_{2p} x_{t-p} + u_{2t}$$

(b) Suppose the two variables are stationary:

(i) Which factors would you take into account in determining the ('optimal') number of lags to include in the model? Which criteria would you use to determine that number? **(3 points)**

(ii) Which method would you use to estimate the model? Why? **(2 points)**

(iii) Explain how you would use the Granger causality test to examine the relationship between the two variables. **(4 points)**

(iv) How useful/helpful is the variance decomposition in uncovering interrelationships among variables in the system? What is (are) the main limitation(s) of the variance decomposition? **(3 points)**