# AFRICAN ECONOMIC RESEARCH CONSORTIUM

## Collaborative Masters Programme in Economics for Anglophone Africa (Except Nigeria)

## JOINT FACILITY FOR ELECTIVES (JFE) 2015

**JUNE – SEPTEMBER**

# ECONOMETRICS THEORY AND PRACTICE I

**First Semester: Final Examination**

**Duration: 3 Hours** | **Date: Monday, August 3, 2015**

**INSTRUCTIONS:**

1. This examination is divided into two sections: **Section I** and **Section II**. There is one question in Section I which is **compulsory**, and four questions in Section II.
2. Answer **QUESTION 1** in **Section I** and **ANY TWO** questions from **Section II**.
3. You are required to answer **THREE** questions in total. Each question carries twenty marks.
4. Relevant formulae are given in Annex 1. You may use an unprogrammable calculator.

## Section I:

### The Question in this Section is Compulsory

### Question 1: [20 marks]

Consider a mixed process ARMA(1,1) model for a particular $y_t$ series expressed as:

$$y_t = c + \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \qquad (1.1)$$

Where $\varepsilon_t \sim (0, \sigma^2)$ is a white noise process.

(a) What is the necessary condition for stationarity of (1.1)? **[2 marks]**

(b) Derive the mean, variance, autocovariance, and autocorrelation expressions for the ARMA process in (1.1). **[8 marks]**

(c) Express the ARMA process given in (1.1) in an MA(∞) process. Show your work. **[4 marks]**

(d) Suppose the coefficients of ARMA(1,1) are given as follows:

$c = 0.6$, $\phi = 0.5$, and $\theta = -0.3$

Find the numerical values of the coefficients of $\varepsilon_{t-1}$, $\varepsilon_{t-2}$, and $\varepsilon_{t-3}$ in the MA(∞) process in (c) above. **[3 marks]**

(e) What is invertibility? And why is it important? **[3 marks]**

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# Section II:

## Answer ANY TWO Questions from this Section

### Question 2: [20 marks]

Consider the regression model:

$$y = X\beta + e \qquad (2.1)$$

where y is T × 1 vector of observations on the dependent variable, y.

X is T × K matrix of observations on non-stochastic and linearly independent regressors

$$X_1, \quad X_2, \quad X_3, \quad \cdots \quad X_K.$$

β is K × 1 vector of unknown parameters $\beta_1, \beta_2, \beta_3, \ldots, \beta_K$, and

e is T × 1 vector of random error terms

The random errors are uncorrelated with mean zero and variance, $\sigma^2$, and independent of $X_k$, $k = 1, 2, \ldots, K$.

(a) In addition to the usual CLRM assumptions, what specific assumptions are needed in order to use maximum likelihood estimation to find the coefficients of (2.1)? **[2 marks]**

(b) Derive the maximum likelihood estimator for β and $\sigma^2$. **[7 marks]**

(c) Explain why the OLS estimator ($\hat{\beta}$) and the ML estimator ($\tilde{\beta}$) are equivalent. **[3 marks]**

(d) Prove that the MLE $\tilde{\sigma}^2$ is biased in small sample but consistent in large sample.

**[4 marks]**

(e) Show that when the classical linear regression model assumptions hold, the method of moments estimation is equivalent to OLS estimator. **[4 marks]**

## Question 3: [20 marks]

Non-linearities enter economic models in various forms. For example, consider the following model:

$$y_t = f(x_t, \beta) + u_t \qquad (3.1)$$

where

$\beta$ is vector of unknown parameters

$f(.)$ is nonlinear function of $\beta$

$x_t$ is vector of regressors.

(a) Differentiate between linear and non-linear models. **[4 marks]**

(b) Given an example of a particular non-linear model. **[3 marks]**

(c) Derive the non-linear least squares estimator for model (3.1) applying the Taylor's series expansion. **[7 marks]**

(d) Describe briefly the steps involve when using an optimization method, e.g. Gauss-Newton method to estimate the unknown coefficients of a non-linear model. **[6 marks]**

## Question 4: [20 marks]

The vector autoregression (VAR) model is popularized by Sim's (1980) influential work. He argued that all variables in an economic system should be treated asymmetrically or all should be treated as endogenous variables. In this way, inappropriate restrictions imposed on a system of equations to achieve identification, can be avoided.

(a) What is a VAR model? **[2 marks]**

(b) Suppose you have $T$ observations on gross domestic product (GDP), aggregate consumption, and private investment — and want to develop a VAR model to relate these variables. If GDP is the first variable in the VAR(p) model, write the specific equation. **[5 marks]**

(c) The number of lags, $p$ in VAR model is usually established empirically. Suggest one method on how to determine this $p$. **[3 marks]**

(d) The vector moving average (VMA) representation is an especial tool to examine interaction among the variables.

Let $y_{1t}$ = GDP; $y_{2t}$ = aggregate consumption; and $y_{3t}$ = private investment. The VMA representation of the VAR(p) model with GDP, aggregate consumption and private investment maybe written as:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} + \sum_{i=1}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) & \phi_{13}(i) \\ \phi_{21}(i) & \phi_{22}(i) & \phi_{23}(i) \\ \phi_{31}(i) & \phi_{32}(i) & \phi_{33}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-i} \\ \varepsilon_{2t-i} \\ \varepsilon_{3t-i} \end{bmatrix}$$

(i) What is the significance of coefficients $\phi_{11}(1)$, $\phi_{12}(1)$, and $\phi_{13}(1)$?

(ii) What is the *n*-period cumulated effects of the shocks in private investment and aggregate consumption on GDP?

**[6 marks]**

(e) The forecast error variance decomposition tells us the proportion of the movements in a sequence due to its "own" shocks versus shocks to the other variables. Based on this, when is a sequence, $y_{1t}$ considered exogenous or endogenous? **[4 marks]**

## Question 5: [20 marks]

Economic theory suggests that similar goods should be close substitutes for each other. The information below shows results from a time series analysis on the prices of regular oranges and organic oranges (grown without chemical pesticides and fertilizers) in a certain market. These are two closely related products, but many consumers are willing to pay somewhat more for organic oranges, perceiving them to be healthier.

As a researcher, analyze the relationship of the prices of these two goods given the following information:

**Figure 5.1 Plot of Prices of Regular and Organic Oranges**

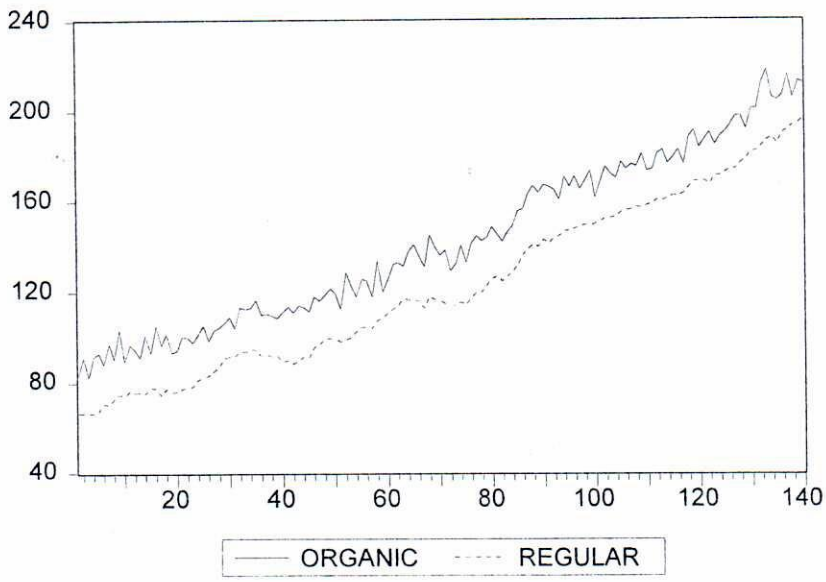

**Table 5.1(a) ADF tests for Prices of Organic Oranges**

| No. of Lags | t-ADF Intercept | AIC | t-ADF Trend and Intercept | AIC |
|---|---|---|---|---|
| Nlag = 0 | -0.7283 | 6.4806 | -6.7606 | 6.2087 |
| Nlag = 1 | 0.1329 | 6.1621 | -3.9656 | 6.0624 |
| Nlag = 2 | 0.4125 | 5.9980 | -2.5337 | 5.9620 |
| Nlag = 3 | 0.6228 | 6.0087 | -2.4942 | 5.9726 |
| Nlag = 4 | 0.7670 | 6.0254 | -2.5321 | 5.9865 |

**Table 5.1(b) ADF tests for Prices of Regular Oranges**

| No. of Lags | t-ADF Intercept | AIC | t-ADF Trend and Intercept | AIC |
|---|---|---|---|---|
| Nlag = 0 | 1.4308 | 3.8130 | -1.5588 | 3.8047 |
| Nlag = 1 | 1.5764 | 3.8175 | -1.2661 | 3.8156 |
| Nlag = 2 | 1.2281 | 3.8216 | -1.5206 | 3.8145 |
| Nlag = 3 | 1.0248 | 3.8376 | -1.6728 | 3.8270 |
| Nlag = 4 | 1.0734 | 3.8590 | -1.7035 | 3.8471 |

ACV = -2.89 at 5% for model with Intercept; and

ACV = -3.45 at 5% for model with Intercept and Trend.

ADF test on residuals (ehat): nlag =4

| | | | |
|---|---|---|---|
| ADF Test Statistic | -4.567514 | 1% Critical Value* | -3.4796 |
| | | 5% Critical Value | -2.8828 |
| | | 10% Critical Value | -2.5780 |

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EHAT)

Sample(adjusted): 6 140
Included observations: 135 after adjusting endpoints

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| EHAT(-1) | -0.834619 | 0.182729 | -4.567514 | 0.0000 |
| D(EHAT(-1)) | -0.238358 | 0.168963 | -1.410711 | 0.1607 |
| D(EHAT(-2)) | -0.146100 | 0.154805 | -0.943768 | 0.3471 |
| D(EHAT(-3)) | 0.041545 | 0.128888 | 0.322334 | 0.7477 |
| D(EHAT(-4)) | 0.038239 | 0.087812 | 0.435460 | 0.6640 |
| C | -0.054104 | 0.334387 | -0.161801 | 0.8717 |

| | | | |
|---|---|---|---|
| R-squared | 0.554304 | Mean dependent var | -0.063810 |
| Adjusted R-squared | 0.537029 | S.D. dependent var | 5.706090 |
| S.E. of regression | 3.882536 | Akaike info criterion | 5.594280 |
| Sum squared resid | 1944.557 | Schwarz criterion | 5.723404 |

ADF test on residuals (ehat): nlag =2

| | | | |
|---|---|---|---|
| ADF Test Statistic | -5.306026 | 1% Critical Value* | -3.4789 |
| | | 5% Critical Value | -2.8825 |
| | | 10% Critical Value | -2.5778 |

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EHAT)

Sample(adjusted): 4 140
Included observations: 137 after adjusting endpoints

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| EHAT(-1) | -0.804821 | 0.151681 | -5.306026 | 0.0000 |
| D(EHAT(-1)) | -0.267582 | 0.125266 | -2.136106 | 0.0345 |
| D(EHAT(-2)) | -0.184685 | 0.085505 | -2.159931 | 0.0326 |
| C | 0.016019 | 0.331050 | 0.048387 | 0.9615 |

| | | | |
|---|---|---|---|
| R-squared | 0.551879 | Mean dependent var | 0.008003 |
| Adjusted R-squared | 0.541771 | S.D. dependent var | 5.721918 |
| S.E. of regression | 3.873314 | Akaike info criterion | 5.574861 |
| Sum squared resid | 1995.341 | Schwarz criterion | 5.660115 |

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Note: Asymptotic critical values for Cointegration test: m = 2
ACV = -3.90 at 1%; -3.34 at 5%; -3.04 at 10%

(a) Analyze the behaviour of the two series as shown in Figure 5.1. **[2 marks]**

(b) Differentiate between stationary and non-stationary variables. **[4 marks]**

(c) Do the prices exhibit stationary behaviour? Support your answer. **[6 marks]**

(d) What is cointegration? **[2 marks]**

(e) Describe the Engle and Granger residual-based test for cointegration. **[2 marks]**

(f) Is there a presence of cointegration between the prices of regular and organic oranges? Support your answer. **[4 marks]**

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# Annex I: Relevant Formulae and Probability Distributions

## Normal Distribution

If $x$ is a normal random variable with mean $= \mu$ , and variance = $\sigma^2$, then

p.d.f. $\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-1}{2\sigma^2} (x-\mu)^2 \right\}$

## Taylor's Series Expansion

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \cdots$$
$$+ \frac{f^n(x_0)}{n!}(x - x_0)^n$$

## Some Matrix Differentiation Rules

1. If $z = a'x = \sum_{i=1}^{N} a_i x_i$ where $a$ and $x$ are column vectors,

   Then $\frac{\partial z}{\partial x} = \frac{\partial (a'x)}{\partial x} = a$

2. If $A$ and $X$ are matrices, and if $A$ is symmetric,

   Then $\frac{\partial\, X'AX}{\partial X} = 2AX$

   If A is not symmetric,

   Then $\frac{\partial X'AX}{\partial X} = (A + A')\, X$