AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative PhD Programme in Economics for Sub-Saharan Africa

COMPREHENSIVE EXAMINATIONS IN CORE AND ELECTIVE FIELDS

FEBRUARY 11 – MARCH 2, 2015

MICROECONOMICS

Time: 08:00 – 11:00 GMT **Date: Monday, February 16, 2015**

INSTRUCTIONS:

Answer a total of FOUR questions: ONE question from Section A, ONE question from Section B, and TWO questions from Section C.

The sections are weighted as indicated on the paper.

SECTION A: (15%)

Answer only ONE Question from this Section

Question 1

(a) Give two examples of public goods and explain how their competitive prices are determined. **[10 Marks]**

(b) Explain why markets under-supply public goods. **[5 Marks]**

Question 2

(a) Explain with illustration, the fact that a decrease in the own price of a normal good will cause quantity demanded to increase. **[7.5 Marks]**

(b) Explain, with illustration the statement that "if own price decrease causes a decrease in quantity demanded, the good must be inferior" [**7.5 Marks]**

SECTION B: (25%)

Answer only ONE Question from this Section

Question 3

(a) Derive the profit function $\pi(P)$ and the supply function (or correspondence) for the single output technology whose production function is $f(z) = \sqrt{(z_1 + z_2)}$ **[12 Marks]**

(b) Derive the cost function $c(w,q)$ and conditional factor demand functions (or correspondences) for the single output constant return technology

$$f(z) = (Z_1^{\rho} + Z_2^{\rho})^{\frac{1}{\rho}}, \rho \leq 1$$

[13 Marks]

Question 4

A study of demand for public transport in a town has yield the following relation:

$$Q^d = 5450 - 2000P - 0{,}1R + 100P_a$$

where Q^d is the number of tickets daily requested; P is the price of the ticket, R is the average income of the users of public transport and P_a is the average cost charged per car covering the same distance.

(i) Give a clear interpretation of the signs (positive or negative) preceding the coefficient related to each explanatory variable in the demand function. **[6 Marks]**

(ii) Derive the demand equation if R=$ 300 and P_a= $ 5.80 and also give a graphical representation. **[4 Marks]**

(iii) What should be the price of the ticket if Q^d=4000? **[2 Marks]**

(iv) If , calculate the demand elasticity and discuss its variation along the demand curve. Q^d=3000 **[8 Marks]**

(v) What would be the number of additional passengers if the average cost of the displacement by car increased by $ 2? **[5 Marks]**

SECTION C: (60%)

Answer TWO Questions from this Section,

Question 5 is COMPULSORY

Question 5 (Compulsory)

Define briefly the **underlined concepts** in the following statements and then **explain** whether the statements are **true** or **false**? **Answer any 4**

(a) **The equilibrium price and quantity** for a market with the following **demand and supply functions**, D(p) = 20-2p and S(p) = 40-6p respectively are 3 and 10. **[7.5 Marks]**

(b) **Equivalent variation** and Compensating Variation are identical for a **quasi-linear utility function**. **[7.5 Marks]**

(c) **Decreasing returns to scale** imply a **linearly homogenous production function**. **[7.5 Marks]**

(d) **Pareto efficiency** always implies **competitive equilibrium**. **[7.5 Marks]**

(e) Any **constant sum game** can be transformed into a **zero sum game**. **[7.5 Marks]**

(f) The **market for lemons** a la Akerlof can be characterized as a game of **asymmetric information**. **[7.5 Marks]**

Question 6

(a) Show that if consumers have strictly increasing and convex preferences and firms have convex production possibility sets, then any Pareto efficient allocation in which all individuals have strictly positive consumption of all goods, can be achieved as the competitive equilibrium generated by some distribution of initial endowments. **[15 Marks]**

(b) Consider the one producer and one consumer Robin Crusoe economy characterized by a firm with the production function, $X = L^{1/2}$, where X is output and L is labor. The consumer buys X with her wage income and the profit she earns from what she produces. Her utility function is:

$U(X,L) = \alpha \ln(X) + (1- \alpha) \ln (1-L)$ where α is a parameter. The price of labor L is w and the price of X is p which is normalized to 1.

(i) Determine the equilibrium wage rate. **[7.5 Marks]**

(ii) Find the Walrasian equilibrium quantities of X and L. **[7.5 Marks]**

Question 7

Firm A and B are the only producers of soft drinks for the market. The inverse market demand curve is given by $P = a - Q$ where Q is the total supply to the market. The price will be zero if $Q > a$. Firm A has constant marginal cost of production c_1, while B's cost of production is c_2. The two firms make their production decisions independently of each other.

(a) Derive the Nash equilibrium of this game. You may assume that the marginal costs are such that an interior solution exists. **[6 Marks]** .

(b) What is the equilibrium level of profit for Firm A arising from this equilibrium? **[6 Marks]**

(c) Assume now that before this interaction Firm A can invest in processes that will affect its marginal cost of production. In particular assume that:

$c_1 = 0.5a - \sqrt{r}$ Where r is the cost of the investment incurred, $r \leq 1/4\, a^2$.

Assuming that the outcome of the second stage is the Nash equilibrium, show that the anticipated profit for Firm A is

$$\left(\frac{1}{3}c_2 + \frac{2}{3}\sqrt{r}\right)^2 - r\,.$$ **[6 Marks]**

(d) Solve for the equilibrium level of investment r, given that a=12 and $c_2 = 6$. **[6 Marks]**

(e) What are the subgame perfect equilibrium strategies of this game? **[6 Marks]**

Question 8

The owner of a firm, *P*, wishes to hire a manager, *M*, for a project in which the profits are affected, to some extent by the manager's effort. The manager has two costly effort levels, e_H (high-effort) and e_L (low-effort). The manager's effort is not perfectly deducible from the observable level of profit, π, which lies between $\underline{\pi}$ and $\overline{\pi}$. The manager receives a wage, *w* from the owner. The manager is an expected utility maximizer with a Bernoulli utility function $u(w, e) = v(w) - g(e)$ over his wage *w* and effort level *e*. Assume that $u_w(w, e) > 0$ and $u_{ww}(w, e) \leq 0$ at all (w, e) (subscripts denote partial derivatives) and $u(w, e_H) < u(w, e_L)$ *at w*. The manager has reservation utility denoted by $\overline{u}$. Use this information to address the following questions;

(a) Explain the meanings of the derivatives and inequalities $u_w(w, e) > 0$, $u_{ww}(w, e) \leq 0$ and $u(w, e_H) < u(w, e_L)$ and how they are related to $v'(w)$ and $g'(e)$. **[5 Marks]**

(b) Give an intuitive explanation of the contracts that the owner might have to offer the manager. **[7 Marks]**

(c) Suppose that the firm's profits are stochastically related to e by the conditional density function $f(\pi \mid e) > 0$ where the distribution of π conditional on e_H first-order stochastically dominates the distribution conditional on e_L (i.e. the level of expected profits when the manager chooses e_H is larger than that from e_L: $\int \pi f(\pi \mid e_H) d\pi > \int \pi f(\pi \mid e_L) d\pi$). Derive and discuss the optimal contract when effort is not observable for both the risk neutral and risk averse manager**.** **[18 Marks]**