



Collaborative MA Programme in Economics for Anglophone Africa  
(Except Nigeria)

JOINT FACILITY FOR ELECTIVES

JUNE – OCTOBER 2008

ECONOMETRICS THEORY & PRACTICE I

First Semester: Final Examination

Time: 09.00 AM – 12.00 Noon

Date: Tuesday, August 12, 2008

**INSTRUCTION:**

Answer any FOUR questions

**Question 1**

- (a) The following model is fit to consumption and income data for Zambia over the period 1985-2003 using PcGive

$$C_t = \alpha + \beta Y_t + \varepsilon_t$$

where  $C$  is natural logarithm of real consumption and  $Y$  is natural logarithm of real GDP and the sequence  $\{\varepsilon_t\}$  is assumed to be white noise. The results are given below

Estimated Coefficients and Standard Errors

	Coefficient	Stand. Error	t-value	t-probability
Constant	1.51	0.44	3.42	0.003
$Y_t$	0.65	0.09	7.01	0.00

Equation standard error ( $\sigma$ ) = 0.057, RSS = 0.0551,  $\hat{R}^2 = 0.74$ ,

$F(2,15) = 49.13$  [0.000]\*\*, DW = 1.57, Number of observations = 19,

Number of parameters = 2, mean ( $C$ ) = 4.61, and var( $C$ ) = 0.0113.

DF statistic for residuals = -3.269 (critical values 5% = -1.96, 1% = -2.71)



Test Summary

Test	Test Statistic	Probability
Autocorrelation	$F(2,15) = 0.23$	0.80
Normality	$\chi^2(2) = 0.91$	0.64
Hetero test	$F(2,14) = 0.74$	0.50
Hetero-X test	$F(2,14) = 0.74$	0.50
RESET	$F(1,16) = 1.31$	0.27

Analyse results output and comment on model adequacy.

(5 marks)

- (b) Univariate processes AR(2) for mean-adjusted consumption ( $c_t = C_t - \bar{C}$ ) and mean-adjusted income ( $y_t = Y_t - \bar{Y}$ ) in Zambia are estimated over the same period using the same data set. The estimated models are reported below

$$c_t = 0.81c_{t-1} - 0.019c_{t-2}$$

$$y_t = 1.05y_{t-1} - 0.26y_{t-2}$$

- (i) Compute autocorrelation function for consumption using Yule-Walker equations for three lags. (3 marks)
- (ii) Show whether the AR processes characterising consumption and income are stationary. (4 marks)
- (iii) What are the implications of your answer in (ii) for estimation results in part (a) above? (3 marks)

## Question 2

- (a) Given the simultaneous equations model

$$y_{1,t} = \alpha_0 + \alpha_1 y_{2,t} + \alpha_2 y_{1,t-1} + u_{1,t}$$

$$y_{2,t} = \beta_0 + \beta_1 y_{1,t} + \beta_2 y_{2,t-1} + \beta_3 y_{2,t-2} + \beta_4 y_{2,t-3} + u_{2,t}$$

where  $y_{1,t}$  and  $y_{2,t}$  are both endogenous variables in period  $t$ , and lagged variables are predetermined or exogenous variables.

- (i) Derive the reduced form model. (3 marks)
- (ii) Give an interpretation of the parameters in the reduced form model you obtained above. (2 marks)



- (b) Given the two-variables, one-lag vector autoregressive (VAR) system below

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} 1.2 & -0.2 \\ 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

where the errors  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$  are white noise and contemporaneously correlated.

- (i) Find the eigenvalues for the given model and determine the rank of the long-run matrix  $\Pi$ , where  $\Pi = I - A$  and  $A$  is the coefficients matrix of the VAR in level. (5 marks)
- (ii) Is the given VAR system cointegrated; i.e., does a vector error correction representation of the VAR in level exist? If the error correction representation exists, use obtained eigenvalues and eigenvectors to find the vector error correction formulation of the VAR. (5 marks)

### Question 3

- (a) Given the autoregressive model

$$Y_t = \alpha + \beta Y_{t-1} + v_t,$$

where  $v_t = \mu_t - \lambda \mu_{t-1}$ ,

- (i) Show that if  $E(\mu_t) = 0$ ,  $E(\mu_t^2) = \sigma^2$ , and  $E(\mu_t \mu_{t-s}) = 0$ ;  $s \neq 0$ , then  $E(Y_{t-1} v_t) \neq 0$ . (5 marks)
- (ii) Show that the ordinary least squares estimator  $\hat{\beta}$  is inconsistent. (5 marks)

- (b) Assess the validity of the statement

*"when common factors are present in polynomials of autoregressive distributed lag (ARDL) model, a static model with autoregressive errors of the order equivalent to the number of common factors existing is the best fit for the data generating process, while in the absence of common factors a dynamic ARDL model that approximates the autoregression in the errors is a best fit for the true data generating process".* (5 marks)



#### Question 4

- (a) In the neoclassical demand for import model (ignoring the price variable)

$$m_t = \alpha m_{t-1} + \beta_0 y_t + \beta_1 y_{t-1} + v_t,$$

where  $m$  and  $y$  are natural logarithms of real imports and real income, respectively.

- (i) Show that the assumption of homogeneity of import demand to income (unitary long-run demand elasticity with respect to income) amounts to an error correction mechanism that entails the process  $m_{t-1} - y_{t-1}$ . What is the economic interpretation of error correction and adjustment processes? **(3 marks)**
- (ii) Since the variance of long-run multiplier or elasticity can not be directly estimated in this model, show how to find the variance of long-run multiplier from variances of parameters in the dynamic model above. (Note that estimation of the given model would give standard errors or variances of the model parameters  $\alpha$ ,  $\beta_0$ , and  $\beta_1$ ). **(3 marks)**
- (iii) If the coefficient  $\beta_1$  in the given model is assumed to equal zero and the resultant model is estimated using OLS to find  $\hat{\alpha}$  and  $\hat{\beta}_0$ . Would the OLS estimate  $\hat{\beta}_0$  be consistent? Explain your answer. **(4 marks)**

- (b) Show that the process:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t$$

where  $\varepsilon_t \sim N(\mu, \sigma^2)$  and  $y_0 = 0$ , is nonstationary when  $\alpha_1 = 1$ . **(5 marks)**

#### Question 5

- (a) The ARMA(1,1) model for the natural logarithm of demand for real balances in Kenya over 1969-2003 is estimated. The estimated model is given by

$$m_t = 0.963 m_{t-1} + \varepsilon_t + 0.121 \varepsilon_{t-1}$$

s.e (0.047)      (0.0) (0.187)



Estimation statistics:

log-likelihood = 27.227, AIC.n = -48.30, AIC = -1.38,  $\hat{\sigma}^2 = 0.011$ , n = sample size (35 observations).

- (i) Use relevant output to compute the process variance and covariance for one lag. (2 marks)
- (ii) Find the moving average representation of the process. (5 marks)
- (b) In a system of simultaneous equations the order and rank conditions may not always *both* be satisfied. Write a model that has the characteristic that the order condition is satisfied but the rank condition is not. Explain your answer. (3 marks)
- (c) Show that the variance of the maximum likelihood estimator for  $\beta$  and  $\sigma^2$  in the linear regression model  $Y = X\beta + \mu$  is given by

$$\begin{bmatrix} \sigma^2(X'X)^{-1} & 0 \\ 0 & \frac{2\sigma^4}{n} \end{bmatrix}$$

(5 marks)

