AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative Masters Programme in Economics for Anglophone Africa (Except Nigeria)

JOINT FACILITY FOR ELECTIVES (JFE) 2017

JUNE – SEPTEMBER

ECONOMETRICS THEORY AND PRACTICE I

First Semester: Final Examination

Duration: 3 Hours **Date: Tuesday, August 7, 2017**

INSTRUCTIONS:

1. This examination consists of two sections: **Section A** and **Section B.**
2. Answer **TWO** questions in **Section A** and **TWO** questions in **Section B.** Note that **Question 1** and **Question 4** are compulsory.
3. You are required to answer a total of **FOUR** questions. All questions carry equal marks.
4. Present your work in a clear and orderly manner.

Section A:

Answer TWO Questions from this Section. Note that Question 1 is Compulsory

Question 1 (Compulsory)

(a) Consider the linear regression model:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$

where $\boldsymbol{y}$ is a N ×1 vector of observations on the dependent variable, $\boldsymbol{X}$ is a N×K matrix of non-stochastic and linearly independent regressors, $\boldsymbol{\beta}$ is a K × 1 vector of unknown parameters, and $\boldsymbol{\varepsilon}$ is a N× 1 vector of random error terms.

(i) In addition to the Classical Linear Regression Model assumptions, what specific assumptions are needed in order to use maximum likelihood estimation to find the coefficients of the equation above? ***(1 mark)***

(ii) Derive the maximum likelihood estimator for $\boldsymbol{\beta}$ ***(5 Marks)***

(iii) Derive the maximum likelihood estimator for $\boldsymbol{\sigma^2}$ ***(3 Marks)***

(iv) Prove that the maximum likelihood estimator for $\boldsymbol{\sigma^2}$ is biased in small samples but consistent in large samples. ***(2 marks)***

(b) State and explain two characteristics of a good estimator. ***(4 marks)***

Question 2

(a) What is Wold Decomposition? ***(1 mark)***

(b) Consider the following second order stochastic difference equation:

$$y_t = 1.5y_{t-1} - 0.5y_{t-2} + \varepsilon_t \quad (1)$$

where ε_t is a white noise process.

(i) Given the initial conditions for y_0 and y_1, find the solution for y_t in terms of the current and past values of the ε_t sequence ***(5 marks)***

(ii) Using the solution in part (i), find the mean and the variance of y_t ***(4 marks)***

(iii) Comment on the stationarity of the y_t sequence ***(1 mark)***

(c) In a study of the demand for money in Sub-Saharan Africa, the following equation was estimated:

$$m_t = u + \emptyset m_{t-1} + \partial_1 I_t + \partial_2 r_{bt} + \partial_3 r_{ct} + \varepsilon_t$$

where;

m_t = Logarithm of real money holdings

I_t = Logarithm of aggregate real income

r_{bt} = Interest rates on bank accounts

r_{ct} = Logarithm of interest rates on commercial paper (T bills and bonds)

ε_t = Error term

Using quarterly data, the estimated function was as follows:

$$m_t = 0.27 + 0.72m_{t-1} + 0.19I_t - 0.45r_{bt} - 0.019r_{ct}$$

What will happen to the money demand two quarters from now, if current income were to increase by one unit with future income I_{t+1} and I_{t+2}unaffected? ***(4 marks)***

Question 3

(a) What do you understand by the terms trend stationary series and difference stationary series? ***(2 marks)***

(b) The following regressions are based on money supply (MS) data for Ghana for the period from 1970 to 2009, for a total of 39 observations. The values in parentheses are t-values, t denotes trend.

$$\Delta MS = 0.503 + 0.0001t - 0.042MS_{t-1} + 0.092\Delta MS_{t-1} + 0.194\Delta MS_{t-2} + v_1$$
$$(2.12)\quad (0.148)\quad (-3.584)\quad (2.009)\quad (4.179)$$

(i)

Sum of Squared Re*siduals*(2042.37)
Durbin – Watson(2.001) *Log Likelihood*(–983.883)
Alkaike inf*o Critern*(4.366) *Schwarz Criterion*(4.411)

$$\Delta MS = 0.527 - 0.041MS_{t-1} + 0.093\Delta MS_{t-1} + 0.193\Delta MS_{t-2} + v_2$$
$$(3.02)\quad (-3.333)\quad (2.007)\quad (4.181)$$

(ii)

Sum of Squared Re*siduals*(2042.49)
Durbin – Watson(2.001) *Log Likelihood*(–983.894)
Alkaike inf*o Critern*(4.361) *Schwarz Criterion*(4.37)

$$\Delta MS_t = -0.013MS_{t-1} + 0.081\Delta MS_{t-1} + 0.193\Delta MS_{t-2} + v_3$$
$$(-1.931)\quad (1.745)\quad (3.858)$$

(iii)

Sum of Squared Re*siduals*(2084.03)
Durbin – Watson(2.003) *Log Likelihood*(–988.456)
Alkaike inf*o Critern*(4.377) *Schwarz Criterion*(4.40)

NB. ***The critical values for ADF at 1% significance level are -3.983, -3.447, and -2.57 for regressions (i), (ii) and (iii), respectively.***

Examining the preceding regressions critically, test for stationarity of the money supply series at 1 % significance level (show the steps clearly). ***(6 marks)***

(c) The equation below models Africa's wages against productivity of workers in the banking sector using current prices. Assume the variables are I(0).

$$W_t = \beta_0 + \beta_1 PROD_t + \varepsilon_t \qquad (2)$$

where W = hourly wage per worker and PROD = labour productivity measured as output per worker.

The estimated regression for the above model is as follows.

Ordinary Least Squares Estimation
****
Dependent variable is W
30 observations used for estimation from 1971 to 2000
****

Regressor	Coefficient	Standard Error	T-Ratio[Prob]
CONST	.50493	.17812	2.8347[.008]
PROD	2.8482	2.7427	1.0382[.307]

****

R-Squared	.29847	R-Bar-Squared	.25170
S.E. of Regression	5.4617	F-stat. F(1, 28)	
Mean of Dependent Variable	7.7479	S.D. of Dependent Variable	6.3138
Residual Sum of Squares	894.9065	Equation Log-likelihood	-101.2785
Akaike Info. Criterion	-104.2785	Schwarz Bayesian Criterion	-106.5232
DW-statistic	0.2145	Durbin's h-statistic	*NONE*

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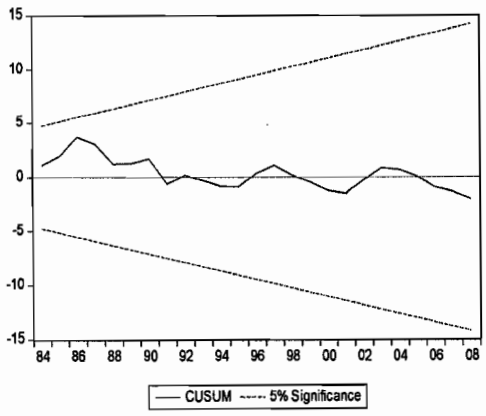
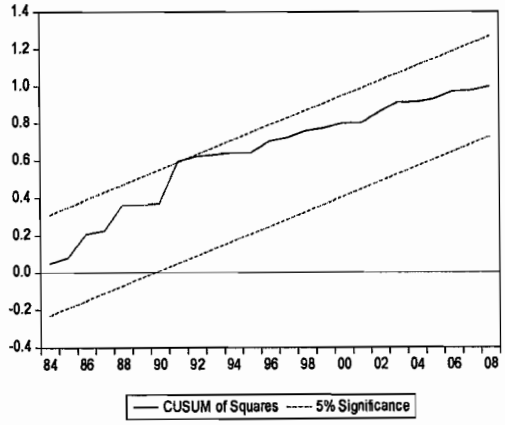
(i) Does the model given by equation (2) suffer from endogeneity/exogeneity problems? Explain. ***(3 marks)***

(ii) Why and how would you attempt to account for business cycles in the model? ***(2 marks)***

(iii) Stating both null and alternative hypothesis, comment on the parameter constancy based on the following graphs showing the cumulative sum of recursive residuals (CUSUM) and cumulative sum of recursive residuals squared (CUSUMSQ). ***(2 marks)***





Section B:

Answer TWO Questions from this Section. Note that Question 4 is Compulsory

Question 4 (Compulsory)

(a) What is a VAR model? *(1 Mark)*

(b) Consider a first order vector autoregression denoted VAR(1) written as:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

$$E(e_t) = 0, E(\varepsilon_t \varepsilon_t') = \begin{cases} \Omega & for\ t \sim \tau \\ 0 & otherwise \end{cases}, \Omega \text{ is symmetric positive definite matrix}$$

Where,

x_t = an (n×1) vector of n variables

A_0 = an (n×1) vector of intercept terms

A_1 = n×n matrices of coefficients

e_t = an (n×1) vector of error terms that may be contemporaneously correlated but are uncorrelated with their own lagged values and uncorrelated with all of the right-hand side variables.

(i) Illustrate that under stationarity, the first order vector autoregression above can be written as a vector moving average *(5 marks)*

(ii) What does the sets of coefficients in the vector moving average represent and measure? *(2 marks)*

(c) For a financial development indicator autoregressive distributed lag model (ARDL), the error correction form is given by:

$$\Delta \log FD_t = b_{10} + \sum_{i=1}^{n} b_{1i} \Delta \log FD_{t-1} + \sum_{i=1}^{n} b_{2i} \Delta logR + \sum_{i=1}^{n} b_{3i} \Delta logGDP + \gamma ECT_{t-1} + u_t$$

where ECT_{t-1} is the lagged error correction term derived from the long-run co-integration model, Δ denotes the first difference operator, FD is a financial development indicator, GDP is economic growth and R is real interest rate. Applying annual data from Kenya National Bureau of Statistics over the period 1972–2008, the following ARDL cointegrating and long run form was estimated:

ARDL Cointegrating And Long Run Form
Dependent Variable: LOG_FD
Selected Model: ARDL(1, 0, 4)

Cointegrating Form

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Δ(LOG_R)	-0.043586	0.052929	-0.823475	0.4180
Δ (LOG_GDP)	-0.482054	0.147765	-3.262303	0.0032
Δ (LOG_GDP(-1))	0.209551	0.155495	1.347638	0.1899
Δ (LOG_GDP(-2))	0.385934	0.187647	2.056698	0.0503
Δ (LOG_GDP(-3))	0.394184	0.246498	1.599134	0.1224
ECT(-1)	-0.539584	0.120832	-4.465573	0.0001

Cointegrating equation = LOG_FD - (-0.1137*LOG_R + 0.0529*LOG_GDP -0.7036)

Long Run Coefficients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG_R	-0.113681	0.057062	-1.992227	0.0574
LOG_GDP	0.052872	0.019341	2.733702	0.0113
C	-0.703648	0.089605	-7.852815	0.0000

(i) What does the γ-coefficient indicate? ***(1 mark)***

(ii) Does the γ-coefficient have the expected sign in the estimated model? ***(1 mark)***

(iii) Interpret the error correction term coefficient for the model ***(1 mark)***

(iv) Does the model demonstrate an error correction mechanism? Justify your answer. ***(2 marks)***

(v) An analyst carried out an ARDL bounds test for the estimated model and obtained the following results:

ARDL Bounds Test
Null Hypothesis: No long-run relationships exist

Test Statistic	Value	K(Restrictions)
F-statistic	4.950297	2
Critical Value Bounds		
Significance	I0 Bound	I1 Bound
10%	2.63	3.35
5%	3.1	3.87
1%	4.13	5

Does the model exhibit a long run relationship? Justify your answer ***(2 marks)***

Question 5

(a) Given

$$\varepsilon_t = v_t\sqrt{\alpha_0 + \alpha_1\varepsilon_{t-1}^2}$$

where v_t is white noise, E(v_t)=0, var(v_t) = σ_v^2 =1

v_t is independent of ε_{t-1}, α_0 and α_1 are constants such that α_0>0 and 0<α_1 > 1

(i) Derive the conditional mean of ε_t ***(2 Marks)***

(ii) Derive the conditional variance. ***(3 Marks)***

(iii) State the conditions necessary for the conditional variance to be positive. ***(1 mark)***

(iv) Suppose the conditions in (iii) are violated, what would be the implication on the variance? ***(1 mark)***

(b) An analyst estimated a Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model of Africa's consumer price index and obtained the following results. *t*-values are in parenthesis. π_t is Africa's consumer price index and h_t is the variance of the white noise disturbance term ε_t.

$$\pi_t = \underset{(1.82)}{0.0050} + \underset{(7.42)}{0.7155}\pi_{t-1} + \varepsilon_t - \underset{(-2.26)}{0.2831}\varepsilon_{t-1} + \underset{(2.67)}{0.2467}\varepsilon_{t-4}$$

$$h_t = \underset{(1.54)}{174E-5} + \underset{(2.14)}{0.2100}\varepsilon_{t-1}^2 + \underset{(3.62)}{0.6410}h_{t-1}$$

R-squared	-0.260	Box-Pierce Q-Statistic:	372.52
Adjusted R-squared	-0.271	Sum squared resid	1.18E+09
ARCH Test			
F-statistic	11.219	Probability	0.001
Obs* R-Squared	10.611	Probability	0.0011

(i) What do the coefficients in the h_t equation imply? Explain. ***(1 mark)***

(ii) The intercept in the h_t equation is not significant at conventional levels. Would you recommend that the term be excluded from the equation? Explain your answer. ***(2 marks)***

(iii) Do the standardized residuals exhibit additional ARCH? Justify your answer. ***(2 marks)***

(iv) What does the answer in (iii) above imply? ***(1 mark)***

(v) Is the mean equation correctly specified? Explain your answer ***(2 marks)***

Question 6

(a) Consider the following cost function of an electricity generating industry estimated using annual data from 1970 to 2004.

$$C = A Y^{\beta_1} P_1^{\beta_2} P_2^{\beta_3} P_3^{\beta_4} u$$

where C is total cost of production, Y is output in kilowatt hours, P_1 is price of labour input, P_2 is price of capital input, P_3 is price of fuel, and u is the disturbance term.

(i) Write out the equation for estimating the above model using OLS. ***(1 mark)***

(ii) What is the economic interpretation of β_1? Explain its expected sign. ***(2 marks)***

(iii) What is the economic interpretation of β_2? ***(1 mark)***

(b) Suppose inflation causes the price of all inputs to increase by the same percentage.

(i) How would you expect this to affect the cost of the industry? ***(1 mark)***

(ii) What restriction would (i) imply? ***(1 mark)***

(c) Explain clearly how the validity of the restriction in (b) can be checked in the model. ***(3 marks)***

(d) A consumption function that has different short- and long-run marginal propensities to consume can be written in the form

$$\ln C_t = \alpha + \beta \ln Y_t + \gamma \ln C_{t-1} + \varepsilon_t ,$$

which is a **distributed lag** model. In this model, the short-run marginal propensity to consume (MPC) (elasticity, since the variables are in logs) is β, and the long-run MPC is $\delta = \beta/(1-\gamma)$.

The estimated equation based on quarterly data on aggregate Africa consumption and disposable personal income for the years 1950 to 2000 is given as:

$$\ln C_t = 0.003142 + 0.07495 \ln Y_t + 0.9246 \ln C_{t-1} + e_t , \; R^2 = 0.999712, \; s = 0.00874.$$

(0.01055) (0.02873) (0.02859)

Estimated standard errors are shown in parentheses. The Estimated Asymptotic Covariance is Est.Asy. Cov $[b, c] = -0.0008207$.

Test the hypothesis that the long-run MPC is greater than or equal to 1($\delta = 1$) ***(6 Marks)***