AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative Masters Programme in Economics for Anglophone Africa (Except Nigeria)

JOINT FACILITY FOR ELECTIVES (JFE) 2016

JUNE – SEPTEMBER

ECONOMETRICS THEORY AND PRACTICE I

First Semester: Final Examination

Duration: 3 Hours **Date: Thursday, August 4, 2016**

INSTRUCTIONS:

1. This examination consists of two sections: **Section A** and **Section B**.
2. Answer **TWO** questions in **Section A** and **TWO** questions in **Section B.** Note that **Question 1** and **Question 4** are compulsory.
3. You are required to answer a total of **FOUR** questions. All questions carry equal marks.
4. Present your work in a clear and orderly manner.

Section A:

Answer TWO Questions from this Section. Note that Question 1 is Compulsory

Question 1 (Compulsory)

Consider the regression model:

$y = X\beta + \varepsilon$,

where y is a N ×1 vector of the dependent variable observations, X is a N×K matrix of the regressors, β is a K × 1vector of unknown parameters, and ε is a N× 1 vector of random error terms.

(a) State the assumptions required in order to use the method of moments approach in estimating the unknown parameters of the regression model. ***(2 marks)***

(b) Show that when the classical linear regression model assumptions hold, the OLS estimator is equal to the method of moment estimation. ***(5 marks)***

(c) Under what condition are the least square estimates more efficient than the method of moments? ***(1 mark)***

(d) Prove that the OLS estimator $\hat{\beta}$ is an unbiased estimator of the true parameter β. ***(4 marks)***

(e) Suppose that a correctly specified regression model would be

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2 + e,$$

where the two parts of **X** have K_1 and K_2 columns, respectively.

Show that $\boldsymbol{\beta}_1$ will be biased if **y is regressed** on $\mathbf{X}_1$. *(3 marks)*

Question 2

Consider the following ARMA (1,0) model

$$y_t = 3.0 + 0.5y_{t-1} + \varepsilon_t, \qquad (1)$$

where ε_t is a white noise process.

(a) Is the y_t in (1) stationary? Justify your answer. *(1 mark)*
(b) Find the mean and the variance of y_t. *(5 marks)*

(c) Find the autocovariance and the autocorrelation functions for y_t *(5 marks)*

(d) What is Wold Decomposition? *(1 mark)*

(e) Derive the MA(∞) of y_t. *(3 marks)*

Question 3

(a) What do you understand by the term stationarity and strict stationarity in a stochastic process? *(3 marks)*

(b) Show how the Dickey-Fuller (DF) unit root test is derived using the random walk process hypothesis. *(5 marks)*

(c) The equation below models Africa's wages against productivity of workers in the Banking sector using current prices. Assume the variables are I(0).

$$W_t = \beta_0 + \beta_1 \, PROD_t + \varepsilon_t \qquad (1)$$

where W = hourly wage per worker and PROD = labour productivity measured as output per worker.

The estimated regression for the above model is as follows:

```
 Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is W
 30 observations used for estimation from 1971 to 2000
*******************************************************************************
 Regressor          Coefficient      Standard Error       T-Ratio[Prob]
 CONST                 .50493           .17812               2.8347[.008]
 PROD                  2.8482          2.7427                1.0382[.307]
*******************************************************************************
 R-Squared                 .29847              R-Bar-Squared                .25170
 S.E. of Regression          5.4617            F-stat.    F( 1,  28)
 Mean of Dependent Variable   7.7479          S.D. of Dependent Variable    6.3138
 Residual Sum of Squares    894.9065         Equation Log-likelihood     -101.2785
 Akaike Info. Criterion     -104.2785        Schwarz Bayesian Criterion  -106.5232
 DW-statistic                 0.2145          Durbin's h-statistic          *NONE*
*******************************************************************************
                    Diagnostic Tests
*******************************************************************************
*    Test Statistics     *              LM Version         *          F Version
*******************************************************************************
*                        *                                 *
* A:Serial Correlation*CHSQ(   1)=   5.4427[.032]    * F(   1,  27)= 5.3943[.037]
*                        *                                 *
* B:Functional Form    *CHSQ(   1)=    1.5370[.0215] *F(   1,  27)= 0.889[.363]
*                        *                                 *
* C:Normality             *CHSQ(   2)=   1.9068[.385]    *  Not applicable
*                        *                                 *
* D:Heteroscedasticity*CHSQ(   1)=   6.0649[.027]    *F(   1,  27)= 7.1161[.019]
*******************************************************************************
  A:Lagrange multiplier test of residual serial correlation
  B:Ramsey's RESET test using the square of the fitted values
  C:Based on a test of skewness and kurtosis of residuals
  D:Based on the regression of squared residuals on squared fitted values
```

(i) Why is the above estimation unsatisfactory? *(2 marks)*

(ii) Would you suggest including a time trend in the model? Explain. *(3 marks)*

(iii) What steps would you take before deciding if the estimated model suffered from pure autocorrelation? *(2 marks)*

Section B:

Answer TWO Questions from this Section. Note that Question 4 is Compulsory

Question 4 (Compulsory)

(a) What is forecast error variance? *(1 mark)*

(b) The vector moving average (VMA) representation examines the interaction between variables. The vector moving average representation of a VAR(1) model with Z_t and Y_t may be written as:

$$\begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}^{(i)} & \phi_{12}^{(i)} \\ \phi_{21}^{(i)} & \phi_{22}^{(i)} \end{bmatrix} \begin{bmatrix} e_{yt-i} \\ e_{zt-i} \end{bmatrix}$$

Derive n-step ahead forecast variance of the y_t sequence only. *(4 marks)*

(c) For a vector autoregressive (VAR) model of order 2, the error correction form is given by:

$$\Delta Y_t = \gamma\beta' Y_{t-1} + \partial + \Gamma_1 \Delta Y_{t-1} + \varepsilon_t ,$$

where Y_t is a vector. For the two-dimensional case $Y_t = (r_t\ b_t)'$, we obtain the following estimation results where the numbers in parenthesis are *t*-values.

$$\begin{pmatrix} \overline{\Delta r_t} \\ \overline{\Delta b_t} \end{pmatrix} = \begin{pmatrix} \underset{(5.01)}{-0.1285} \\ \underset{(0.18)}{0.0069} \end{pmatrix} \left(r_{t-1} - \underset{(34.29)}{1.1169} b_{t-1} \right) + \begin{pmatrix} \underset{(-4.80)}{-0.1027} \\ \underset{-0.25)}{-0.0080} \end{pmatrix} + \begin{pmatrix} \underset{(4.26)}{0.2716} & \underset{(3.91)}{0.2155} \\ \underset{(0.40)}{0.0389} & \underset{(5.33)}{0.4466} \end{pmatrix} \begin{pmatrix} \Delta r_{t-1} \\ \Delta b_{t-1} \end{pmatrix}$$

(i) What does the γ-coefficient determine? *(1 mark)*

(ii) Does the γ-coefficient have the expected sign in the estimated models? *(1 mark)*

(iii) Interpret the γ-coefficient for the $\overline{\Delta r_t}$ model. *(1 mark)*

(iv) Interpret the longrun cointegrating coefficient for the $\overline{\Delta r_t}$ model. *(1 mark)*

(v) Which of the two estimated models demonstrates error correction mechanism? Justify your answer. *(2 marks)*

(d) An analyst carries out a weak exogeneity test for the estimated models and obtains the following results.

Variable	r_t	b_t
Chi-square -$X^2(1)$	18.100	0.5979
p-value	0.0002	0.4466

(i) What does the term weakly exogenous imply? *(1 mark)*

(ii) Which of the variable is weakly exogenous? Justify your answer. *(2 marks)*

(iii) What is the practical importance of a weakly exogenous variable? *(1 mark)*

Question 5

(a) Given $\varepsilon_t = v_t\sqrt{\alpha_0 + \alpha_1\varepsilon_{t-1}^2}$

where v_t is white noise, E(v_t)=0, var(v_t) = σ_v^2 =1,

v_t is independent of ε_{t-1}, α_0 and α_1 are constants such that α_0>0 and 0<α_1 > 1.

(i) Derive the unconditional expectation of ε_t. *(2 marks)*

(ii) Derive the unconditional variance. *(3 marks)*

(b) The equations below model the logarithmic change of Africa's stock exchange composite index. h_t is the variance of the white noise disturbance term ε_t. I_{t-1} is a dummy variable that takes the value of 1 if $\varepsilon_{t-1}\angle 0$, 0 if $\varepsilon_{t-1} \geq 0$. No's in Parenthesis are t- values

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \lambda_1 I_{t-1}\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad (1)$$

$$ln(h_t) = \alpha_0 + \alpha_1(\varepsilon_{t-1}/h_t^{0.5}) + \lambda_1\left|\varepsilon_{t-1}/h_t^{0.5}\right| + \beta_1\, ln(h_{t-1}) \qquad (2)$$

The estimated regression for the above models are as follows.

$$h_t = \underset{(6.48)}{1.81E-6} - \underset{(-0.662)}{0.0064}\,\varepsilon_{t-1}^2 + \underset{(12.33)}{0.1947}\, I_{t-1}\varepsilon_{t-1}^2 + \underset{(96.85)}{0.8937}\, h_{t-1} \qquad (1a)$$

$$ln(h_t) = \underset{(-9.61)}{-0.4321} - \underset{(-13.71)}{0.1553}(\varepsilon_{t-1} / h_t^{0.5}) + \underset{(7.72)}{0.1263}\left|\varepsilon_{t-1} / h_t^{0.5}\right| + \underset{(230.71)}{0.9646}\, ln(h_{t-1}) \qquad (2a)$$

(i) Why is Model 1 unsatisfactory? ***(1 mark)***

(ii) Is it possible to re-estimate Model 1 without the variable ε_{t-1}^2? Explain ***(2 marks)***

(iii) What does the value of λ_1 coefficient in Model 1 imply? ***(2 marks)***

(iv) What would you conclude about the data based on model 1 results? Explain ***(2 marks)***

(v) Is model 2 satisfactory? Explain ***(2 marks)***

(vi) An analyst found that the test restriction $\alpha_1 + \lambda_1 = 0$ in Model 2 had a *probability-*value of 0.991. What does this imply? ***(1 mark)***

Question 6

(a) Suppose you are given the following two equations:

Equation (1): $INF_t = \beta_1 + \beta_2 M_t + \beta_3 M_{t-1} + \beta_4 M_{t-2} + \varepsilon_t$

Equation (2): $INF_t = \beta_1 + \beta_2 M_t + \varepsilon_t$,

where, "INF" denotes the inflation rate and "M" denotes the growth rate of the money supply.

(i) Which of the above equations is unrestricted? ***(1 mark)***

(ii) State the restrictions to be imposed in model selection. Clearly indicate your null and alternative hypothesis. ***(2 marks)***

(iii) Suppose both models have been estimated on 60 observations and the sum of squared residuals for equation (1) was found to be 224 while the sum of squared residuals for equation (2) was found to be 192. Derive the numerical value of the test statistic for the restrictions in (ii) above. ***(3 marks)***

(iv) Based on (iii), would you accept or reject the null hypothesis in (ii) at 5% significance? Describe in words what your conclusion from this is. ***(3 marks)***

(b) A consumption function that has different short- and long-run marginal propensities to consume can be written in the form

$\ln C_t = \alpha + \beta \ln Y_t + \gamma \ln C_{t-1} + \varepsilon_t$, which is a **distributed lag** model. In this model, the short-run marginal propensity to consume (MPC) (elasticity, since the variables are in logs) is β, and the long-run MPC is $\delta = \beta/(1-\gamma)$.

The estimated equation based on quarterly data on aggregate Africa consumption and disposable personal income for the years 1950 to 2000 is given as:

$$\ln C_t = \underset{(0.01055)}{0.003142} + \underset{(0.02873)}{0.07495} \ln Y_t + \underset{(0.02859)}{0.9246} \ln C_{t-1} + e_t, \quad R^2 = 0.999712, \; s = 0.00874.$$

Estimated standard errors are shown in parentheses. The Estimated Asymptotic Covariance is Est.Asy. Cov $[\beta, \gamma] = -0.0008207$.

Test the hypothesis that the long-run MPC is greater than or equal to 1 ($\delta = 1$).

~~at 5%~~ at 5% significance level. *(6 marks)*

TABLE E The Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

TABLE F The t-Distribution

	0.900	0.700	0.500	0.300	0.200	0.100	0.050	0.020	0.010	α value	two tailed-test
	0.100	0.300	0.500	0.700	0.800	0.900	0.950	0.980	0.990	CL	
	0.450	0.350	0.250	0.150	0.100	0.050	0.025	0.010	0.005	α value	one tailed-test
	0.550	0.650	0.750	0.850	0.900	0.950	0.975	0.990	0.995	CL	
d.f.	Values of t										
1	0.158	0.510	1.000	1.963	3.078	6.314	12.706	31.821	63.657		
2	0.142	0.445	0.816	1.386	1.886	2.920	4.303	6.965	9.925		
3	0.137	0.424	0.765	1.250	1.638	2.353	3.182	4.541	5.841		
4	0.134	0.414	0.741	1.190	1.533	2.132	2.776	3.747	4.604		
5	0.132	0.408	0.727	1.156	1.476	2.015	2.571	3.365	4.032		
6	0.131	0.404	0.718	1.134	1.440	1.943	2.447	3.143	3.707		
7	0.130	0.402	0.711	1.119	1.415	1.895	2.365	2.998	3.499		
8	0.130	0.399	0.706	1.108	1.397	1.860	2.306	2.896	3.355		
9	0.129	0.398	0.703	1.100	1.383	1.833	2.262	2.821	3.250		
10	0.129	0.397	0.700	1.093	1.372	1.812	2.228	2.764	3.169		
11	0.129	0.396	0.697	1.088	1.363	1.796	2.201	2.718	3.106		
12	0.128	0.395	0.695	1.083	1.356	1.782	2.179	2.681	3.055		
13	0.128	0.394	0.694	1.079	1.350	1.771	2.160	2.650	3.012		
14	0.128	0.393	0.692	1.076	1.345	1.761	2.145	2.624	2.977		
15	0.128	0.393	0.691	1.074	1.341	1.753	2.131	2.602	2.947		
16	0.128	0.392	0.690	1.071	1.337	1.746	2.120	2.583	2.921		
17	0.128	0.392	0.689	1.069	1.333	1.740	2.110	2.567	2.898		
18	0.127	0.392	0.688	1.067	1.330	1.734	2.101	2.552	2.878		
19	0.127	0.391	0.688	1.066	1.328	1.729	2.093	2.539	2.861		
20	0.127	0.391	0.687	1.064	1.325	1.725	2.086	2.528	2.845		
21	0.127	0.391	0.686	1.063	1.323	1.721	2.080	2.518	2.831		
22	0.127	0.390	0.686	1.061	1.321	1.717	2.074	2.508	2.819		
23	0.127	0.390	0.685	1.060	1.319	1.714	2.069	2.500	2.807		
24	0.127	0.390	0.685	1.059	1.318	1.711	2.064	2.492	2.797		
25	0.127	0.390	0.684	1.058	1.316	1.708	2.060	2.485	2.787		
26	0.127	0.390	0.684	1.058	1.315	1.706	2.056	2.479	2.779		
27	0.127	0.389	0.684	1.057	1.314	1.703	2.052	2.473	2.771		
28	0.127	0.389	0.683	1.056	1.313	1.701	2.048	2.467	2.763		
29	0.127	0.389	0.683	1.055	1.311	1.699	2.045	2.462	2.756		
30	0.127	0.389	0.683	1.055	1.310	1.697	2.042	2.457	2.750		
40	0.126	0.388	0.681	1.050	1.303	1.684	2.021	2.423	2.704		
60	0.126	0.387	0.679	1.045	1.296	1.671	2.000	2.390	2.660		
120	0.126	0.386	0.677	1.041	1.289	1.658	1.980	2.358	2.617		
∞	0.126	0.385	0.674	1.036	1.282	1.645	1.960	2.326	2.576		

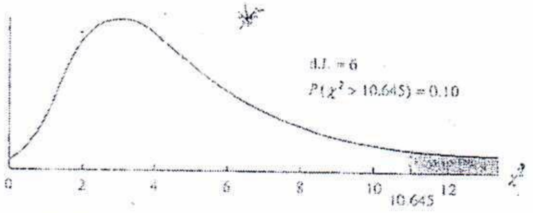

Table II Chi-Square Distribution

d.f.	$\chi^2_{0.995}$	$\chi^2_{0.990}$	$\chi^2_{0.975}$	$\chi^2_{0.950}$	$\chi^2_{0.900}$	$\chi^2_{0.700}$	$\chi^2_{0.500}$	$\chi^2_{0.300}$	$\chi^2_{0.200}$	$\chi^2_{0.100}$	$\chi^2_{0.050}$	$\chi^2_{0.020}$	$\chi^2_{0.010}$
1	0.000	0.000	0.001	0.004	0.016	0.148	0.455	1.074	1.642	2.706	3.841	5.412	6.635
2	0.010	0.020	0.051	0.103	0.211	0.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210
3	0.072	0.115	0.216	0.352	0.584	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.345
4	0.207	0.297	0.484	0.711	1.064	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277
5	0.412	0.554	0.831	1.145	1.610	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086
6	0.676	0.872	1.237	1.635	2.204	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812
7	0.989	1.239	1.690	2.167	2.833	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475
8	1.344	1.646	2.180	2.733	3.490	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090
9	1.735	2.088	2.700	3.325	4.168	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666
10	2.156	2.558	3.247	3.940	4.865	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209
11	2.603	3.053	3.816	4.575	5.578	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725
12	3.074	3.571	4.404	5.226	6.304	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217
13	3.565	4.107	5.009	5.892	7.042	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688
14	4.075	4.660	5.629	6.571	7.790	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141
15	4.601	5.229	6.262	7.261	8.547	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578
16	5.142	5.812	6.908	7.962	9.312	12.624	15.338	18.418	20.465	23.542	26.296	29.633	32.000
17	5.697	6.408	7.564	8.672	10.085	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409
18	6.265	7.015	8.231	9.390	10.865	14.440	17.338	20.601	22.760	25.989	28.869	32.346	34.805
19	6.844	7.633	8.907	10.117	11.651	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191
20	7.434	8.260	9.591	10.851	12.443	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566
21	8.034	8.897	10.283	11.591	13.240	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932
22	8.643	9.542	10.982	12.338	14.041	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289
23	9.260	10.196	11.689	13.091	14.848	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638
24	9.886	10.856	12.401	13.848	15.659	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980
25	10.520	11.524	13.120	14.611	16.473	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314
26	11.160	12.198	13.844	15.379	17.292	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642
27	11.808	12.879	14.573	16.151	18.114	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963
28	12.461	13.565	15.308	16.928	18.939	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278
29	13.121	14.256	16.047	17.708	19.768	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588
30	13.787	14.953	16.791	18.493	20.599	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892
40	20.707	22.164	24.433	26.509	29.051	34.872	39.335	44.165	47.269	51.805	55.758	60.436	63.691
50	27.991	29.707	32.357	34.764	37.689	44.313	49.335	54.723	58.164	63.167	67.505	72.613	76.154
60	35.534	37.485	40.482	43.188	46.459	53.809	59.335	65.227	68.972	74.397	79.082	84.580	88.379
70	43.275	45.442	48.758	51.739	55.329	63.346	69.334	75.689	79.715	85.527	90.531	96.388	100.425

TABLE G The *F*-Distribution (*continued*)

$F_{0.90}$; $\alpha = 0.10$

Denominator degrees of freedom	Numerator degrees of freedom 10	12	15	20	24	30	40	60	120	∞
28	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

$F_{0.95}$; $\alpha = 0.05$

Denominator degrees of freedom	Numerator degrees of freedom 1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30

TABLE G The F-Distribution (*continued*)

Denominator degrees of freedom	$F_{0.95}; \alpha = 0.05$								
	Numerator degrees of freedom								
	1	2	3	4	5	6	7	8	9
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
∞	3.84	3.00	2.61	2.37	2.21	2.10	2.01	1.94	1.88

Denominator degrees of freedom	$F_{0.95}; \alpha = 0.05$									
	Numerator degrees of freedom									
	10	12	15	20	24	30	40	60	120	∞
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81

TABLE G The F-Distribution (*continued*)

Denominator degrees of freedom	$F_{0.95}; \alpha = 0.05$					
	Numerator degrees of freedom					
	10	12	15	20	24	30
23	2.27	2.20	2.13	2.05	2.01	[illegible]
24	2.25	2.18	2.11	2.03	1.98	[illegible]
25	2.24	2.16	2.09	2.01	1.96	[illegible]
26	2.22	2.15	2.07	1.99	1.95	[illegible]
27	2.20	2.13	2.06	1.97	1.93	[illegible]
28	2.19	2.12	2.04	1.96	1.91	[illegible]
29	2.18	2.10	2.03	1.94	1.90	[illegible]
30	2.16	2.09	2.01	1.93	1.89	[illegible]
40	2.08	2.00	1.92	1.84	1.79	[illegible]
60	1.99	1.92	1.84	1.75	1.70	[illegible]
120	1.91	1.83	1.75	1.66	1.61	[illegible]
∞	1.83	1.75	1.67	1.57	1.52	[illegible]

Denominator degrees of freedom	$F_{0.975}; \alpha = 0.025$					
	Numerator degrees of freedom					
	1	2	3	4	5	6
1	647.8	799.5	864.2	899.6	921.8	[illegible]
2	38.51	39.00	39.17	39.25	39.30	[illegible]
3	17.44	16.04	15.44	15.10	14.88	[illegible]
4	12.22	10.65	9.98	9.60	9.36	[illegible]
5	10.01	8.43	7.76	7.39	7.15	[illegible]
6	8.81	7.26	6.60	6.23	5.99	[illegible]
7	8.07	6.54	5.89	5.52	5.29	[illegible]
8	7.57	6.06	5.42	5.05	4.82	[illegible]
9	7.21	5.71	5.08	4.72	4.48	[illegible]
10	6.94	5.46	4.83	4.47	4.24	[illegible]
11	6.72	5.26	4.63	4.28	4.04	[illegible]
12	6.55	5.10	4.47	4.12	3.89	[illegible]
13	6.41	4.97	4.35	4.00	3.77	[illegible]
14	6.30	4.86	4.24	3.89	3.66	[illegible]
15	6.20	4.77	4.15	3.80	3.58	[illegible]
16	6.12	4.69	4.08	3.73	3.50	[illegible]
17	6.04	4.62	4.01	3.66	3.44	[illegible]
18	5.98	4.56	3.95	3.61	3.38	[illegible]
19	5.92	4.51	3.90	3.56	3.33	[illegible]