AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative MA Programme in Economics for Anglophone Africa (Except Nigeria)

JOINT FACILITY FOR ELECTIVES (JFE)

JULY – OCTOBER 2006

ECONOMETRICS THEORY & PRACTICE I

First Semester: Final Examination

Duration: 3 Hours **Date: Wednesday, August 16, 2006**

INSTRUCTION:

Answer **ANY FOUR (4)** Questions

Question 1.

(a) Consider ownership of a car a random variable y which follows a binomial distribution. A sample of *n* individuals is drawn from this distribution, where y takes on the values of zero or one according to the probability density function

$$f(y_i) = \theta^{y_i}(1-\theta)^{(1-y_i)} \;;\; 0 \le \theta \le 1,\; y_i = 0,1 \text{ for all } i = 1, 2, ..., n$$

Thus the probability of owning a car “success, y=1” is given by $f(1) = \theta$, and the probability of not owning a car “failure, y=0” is given by $f(0) = 1-\theta$. Find the maximum likelihood estimator of θ. **(8 points)**

(b) Write short note on the generalised method of moments (GMM) noting how the method is employed to estimate unknown population parameters. **(7 points)**

Question 2.

(a) Consider the model

$$y_{1t} = \alpha_1 + \alpha_2 x_t + \varepsilon_{1t}$$

$$y_{2t} = \beta_1 + \beta_2 x_t + \beta_3 y_{1t} + \varepsilon_{2t}$$

(i) Suppose that

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \right).$$

Which estimator should be used to get consistent parameters? Explain.

(ii) Now suppose that

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix} \right).$$

Which estimator should be used to get consistent parameters? Explain. **(5 points)**

(b) Show that in a simultaneous equation model of G endogenous variables and K exogenous variables satisfaction of the rank condition in determining identification of equations implies satisfaction of the order condition. **(5 points)**

(c) In the context of simultaneous-equations modelling assess the validity of the following statement "*the endo-exogenous division of the variables is theory-informed rather than data-determined*". **(5 points)**

Question 3.

(a) Consider the autoregressive distributed lag model, ARDL (3,3)

$$A(L)y_t = B(L)x_t + \varepsilon_t \ ,$$

where ε_t is a white-noise process.

Show that presence of two common factors $\lambda_1 = \delta_1 = \rho_1$ and $\lambda_2 = \delta_2 = \rho_2$ - where and λ_2 are roots for the polynomial $A(L)$ and δ_1 and δ_2 are roots for the polynomial $B(L)$ – results in a restricted ARDL model of the form

$$\overset{*}{A}(L)y_t = \overset{*}{B}(L)x_t + \mu_t$$

where the errors μ_t follow autoregressive process of order two, AR (2).

(8 points)

(b) Describe the necessary steps you should follow to run simple static and dynamic forecasting using the following estimated ARDL (1,1) model

$$y_t = \widehat{const.} + \hat{\alpha} y_{t-1} + \hat{\beta}_0 x_t + \hat{\beta}_1 x_{t-1}$$

(7 points)

Question 4.

(a) Explain the invertibility condition in moving average time series and show in details which of the following MA (2) processes are invertible:

(1) $y_t = \varepsilon_t - 0.9\varepsilon_{t-1} + 0.2\varepsilon_{t-2}.$

(2) $y_t = \varepsilon_t - 1.8\varepsilon_{t-1} + 0.4\varepsilon_{t-2}.$

(3) $y_t = \varepsilon_t - 0.8\varepsilon_{t-1} + 0.4\varepsilon_{t-2}.$

(6 points)

(b) Define strict stationarity and weak (covariance) stationarity and show the difference between the two concepts, and then show which of the following AR (2) series satisfy the weak stationarity requirements (explain):

(1) $y_t = 0.9y_{t-1} - 0.2y_{t-2} + \varepsilon_t$.

(2) $y_t = 0.8y_{t-1} + 0.4y_{t-2} + \varepsilon_t$.

(3) $y_t = 1.0y_{t-1} - 0.8y_{t-2} + \varepsilon_t$.

(6 points)

(c) Derive the autocorrelation function $\rho(k)$ for the AR(2)

$$y_t = 0.9y_{t-1} - 0.2y_{t-2} + \varepsilon_t$$

using Yule-Walker equations for $k = 0, 1, \cdots, 4$. **(3 points)**

Question 5.

(a) Write down a two-equation vector autoregressive (VAR) model and show how you would state the vector moving average (VMA) representation of the model. Derive the impulse functions and describe their use in policy analysis. **(5 points)**

(b) Given the 2.2 singular matrix $A = \begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}$, use the characteristic equation to find the eigenvalues or characteristic roots λ_i. Also find the characteristic vectors or eigenvectors that correspond to λ_i. **(5 points)**

(c) Explain the concept of cointegration and show how Johansen procedure is applied in cointegration analysis to determine the number of cointegrating vectors. **(5 points)**