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AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative Masters Programme in Economics for Anglophone Africa (Except Nigeria)

JOINT FACILITY FOR ELECTIVES (JFE) 2018

JUNE – SEPTEMBER

ECONOMETRICS THEORY AND PRACTICE I

First Semester: Final Examination

Duration: 3 Hours **Date: Friday, August 10, 2018**

INSTRUCTIONS:

1. This examination consists of two sections: **Section A** and **Section B.**
2. You are required to answer **TWO** questions in **Section A** and **TWO** questions in **Section B.** Note that **Question 1** and **Question 4** are compulsory.
3. All questions carry equal marks.
4. Present your work in a clear and orderly manner.

Section A:

Answer TWO Questions from this Section. Note that Question 1 is Compulsory

Question 1(Compulsory)

Consider a mixed process ARMA (1,1) model for particularly y_t series expressed as:

$$y_t = c + \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1} \quad (1)$$

Where $\varepsilon_t \sim (0, \sigma^2)$ is a white noise process.

(i) What is the necessary condition for stationarity of equation (1). **[2 marks]**

(ii) Derive the autocovariance and the autocorrelation expressions for the ARMA process in (1). **[6 marks]**

(iii) Express the ARMA process in (1) as MA(∞) process. Show your work. **[4 marks]**

(iv) Suppose the coefficients of ARMA(1,1) are given as follows:

$c = 0.6$, $\phi = 0.5$, and $\theta = -0.3$

Find the numerical values of the coefficients of ε_{t-1}, ε_{t-2}, and ε_{t-3} in the process in equation (1) , above. **[3 Marks]**

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Question 2

Non-linearities enter economic models in various forms. For example, consider the following model:

$$y_t = f(\boldsymbol{x}_t, \boldsymbol{\beta}) + \varepsilon_t \quad (2)$$

where:

$\boldsymbol{\beta}$ is a vector of unknown parameters

f(.) is a nonlinear function of $\boldsymbol{\beta}$

$\boldsymbol{x}_t$ is a vector of regressors

(i) Differentiate between linear and non-linear models **[2 Marks]**

(ii) Give an example of a particular non-linear model **[2 Marks]**

(iii) Derive the non-linear least square estimator for model (2) applying the Taylor's series expansion **[6 Marks]**

(iv) Describe briefly the steps involved when using an optimization method, e.g. Gauss Newton method to estimate the unknown coefficients of a non-linear model. **[5 Marks]**

Question 3

(a) Show that the random walk process is an integrated series **[3 Marks]**

(b) The following results were obtained from Clemente-MontaÒÈs-Reyes unit-root test with double mean shifts, Innovative Outlier (IO model).

LogGDP T = 33 optimal breakpoints : 1985 , 1989

AR(0)	*du1*	*du2*	*(rho – 1)*	*constant*
Coefficients:	*0.02983*	*0.05364*	*-0.06999*	*0.38619*
t-statistics:	*1.799*	*2.936*	*-4.464*	
P-values:	*0.082*	*0.006*	*-5.490 (5% crit. value)*	

Does GDP exhibit stationary behavior? Support your answer. **[2 Marks]**

(c) Consider the following second order stochastic difference equation:

$$y_t = 1.5y_{t-1} - 0.5y_{t-2} + \varepsilon_t \quad (3.1)$$

where ε_t is a white noise process.

Explain why it is not possible to obtain the backward looking solution for y_t unless the initial conditions for y_0 and y_1, are given. **[1 Mark]**

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(d) The solution for y_t in (3.1) in terms of the current and past values of the ε_t sequence is given as follows:

$y_t = \sum_{i=0}^{t-2} \propto_i \varepsilon_{t-i} + \propto_{t-1} y_1 + \propto_t y_0$

(i) Find the mean of y_t and comment on the stationarity of the y_t sequence. **[3 Marks]**

(ii) Find the forecast function y_{t+s} **[1 Mark]**

(e) In a study of early retirements from the civil service, the following equation was estimated using census data for 1979 from 44 administrative regions in Kenya:

RETRD=β_0 +β_1HLTH+ β_2MSSEC+ β_3MPUBAS+β_4UNEM +β_5DEP +β_6INDIG +u (3.2)

where:

RETRD = Percent of retired person who are between the ages of 16 and 65

HLTH = Percent of people between 16 and 64 years of age who are prevented from working due to disability

MSSEC = Mean social security income (Kenya Pounds)

MPUBAS = Mean public assistance (welfare) income (Kenya Pounds)

UNEM = Unemployment rate in percent

DEP = Percent of households that are composed of married couples with children under 18

INDIG = Percent of people who are non-British

Using census data, the coefficients were estimated as follows:

Variable	Co-efficient	Std.Error
RETRD	-3.930	9.202
HLTH	1.627379	0.300
MSSEC	-5.483e-04	0.0021
MPUBAS	4.568e-04	0.0017
UNEM	0.549	0.250
DEP	0.153	0.097
INDIG	0.077	0.034
n	44	
SSR	175.08805	

A second model was estimated after omitting MSSEC and MPUBAS and the new SSR is 175.52381. Test the null hypothesis $\beta_2 = \beta_3 = 0$ at the 1 percent level. (NB: Be sure to state the

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alternative hypothesis, the test statistic to compute, its distribution under the null including the degrees of freedom, and the criterion for rejection of the null). **[5 Marks]**

Section B:

Answer TWO Questions from this Section. Note that Question 4 is Compulsory

Question 4 (Compulsory)

(a) What is an Impulse Response Function (IRF)? **[1 Mark]**

(b) The vector moving average (VMA) representation is an essential tool to examine interaction among the variables. Letting Y_t to be Consumption and Z_t to be income, the VMA representation of VAR(1) model may be written as:

$$\begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}^{(i)} & \phi_{12}^{(i)} \\ \phi_{21}^{(i)} & \phi_{22}^{(i)} \end{bmatrix} \begin{bmatrix} e_{yt-i} \\ e_{zt-i} \end{bmatrix}$$

(i) Derive the n-step ahead forecast error of Y_t sequence. **[3 Marks]**

(ii) Derive the forecast error variance of (i) above and decompose the variance into the proportions due to each shocks of the Y_t sequence. **[4 Marks]**

(c) An economist is interested in testing the direction of causation between Africa's Financial deepening (M2GDP) and Interest rate (R) over 1986 to 2018 inclusive. The Granger causality Wald tests to detect the direction of causation led to the following results:

Equation	Excluded	chi2	df	Prob > chi2
LogM2GDP	LogR	5.7455	4	0.219
LogM2GDP	ALL	5.7455	4	0.219
LogR	LogM2GDP	13.917	4	0.008
LogR	ALL	13.917	4	0.008

Using the results in the above table, explain the direction of causation between Financial deepening and Interest rate. **[2 Marks]**

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(d) The following regression results were obtained from the data for the period 1962:I to 2006:IV for East African Community. The values in parenthesis are t-values. M1 represents money supply while e the residual.

Equation (1)

$$\text{LogMI}_t = 60.36 + 0.608\text{LogGDP}_t$$
$$(15.90) \qquad (3.67)$$

R-Squared 0.99 Durbin Watson Statistic 2.03

Equation (2)

$$\Delta\hat{e}_t = -0.301\hat{e}_{t-1} - 0.04\Delta\hat{e}_{t-1}$$
$$(-0.44) \qquad (-1.68)$$

Log likelihood −988.46 Durbin Watson Statistic 2.03

Alkaike Criterion 4.37 Schwarz Criterion 4.4

(i) Is the regression in equation (1) a short-or long-run model? Explain. **[2 Marks]**

(ii) Do you suspect that equation (1) is spurious based on the results for equation(2)? Explain. **[3 Marks]**

Question 5

(a) Suppose that the $\{\varepsilon_t\}$ sequence is the ARCH(q) process is given as follows

$$\varepsilon_t = v_t(\alpha_0 + \alpha_1(\varepsilon_{t-1})^2 +, \ldots, +\alpha_q(\varepsilon_{t-q})^2)^{\frac{1}{2}}$$

where v_t is white noise, E(v_t)=0, var(v_t) = σ_v^2 =1

Derive the conditional variance ($E_{t-1}\varepsilon_t^2$) **[3 Marks]**

(b) Consider the following set of equations that constitute the basic autoregressive conditional heteroskedastic in the mean (ARCH-M) model, used to study asset markets:

$y_t = \mu_t + \varepsilon_t$

$\mu_t = \beta + \delta h_t$ and $h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$

where:

y_t = excess return from holding a long-term asset relative to a one-period treasury bill

μ_t = risk premium necessary to induce the risk-averse agent to hold the long-term asset rather than the one-period bond

ε_t= unforecastable shock to the excess return on the long-term asset

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h_t= conditional variance of ε_t

ε_t = white noise disturbance and $E(\varepsilon_t)^2 = E(\varepsilon_{t-1})^2 = \cdots = 1$

(i) Find the unconditional mean (Ey_t). **[4 Marks]**

(ii) How does a one unit change in δ affect the mean **[1 Mark]**

(c) The following results were obtained from Generalised Autoregressive Conditional Heteroskedasticity (GARCH (1, 1)) model with no regressors in the mean and variance equations.

Dependent Variable: Nairobi Stock Exchange INDEX
Method: ML - ARCH (BHHH) - Normal distribution
Convergence achieved after 47 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	355.3521	13.45119	26.41789	0.0000
	Variance Equation			
C	1411.084	892.2626	1.581467	0.1138
RESID(-1)^2	1.015971	0.175834	5.778026	0.0000
GARCH(-1)	0.066795	0.081337	0.821215	0.4115

R-squared	-0.259593	Mean dependent var	1447.585
Adjusted R-squared	-0.270764	S.D. dependent var	1437.034
S.E. of regression	1619.942	Akaike info criterion	15.49030
Sum squared resid	1.18E+09	Schwarz criterion	15.53551
Log likelihood	-3526.789	Durbin-Watson stat	0.009773
Box-Pierce Q-Statistic:	372.52		
ARCH Test			
F-statistic	11.219	Probability	0.001
Obs* R-Squared	10.611	Probability	0.0011

(i) Do the standardized residuals exhibit additional ARCH? Justify your answer. **[1 Mark]**

(ii) What does the answer in (i) above imply? **[1 Mark]**

(iii) Is the mean equation correctly specified? Explain your answer **[2 Marks]**

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(d) The following correlogram was generated from residuals obtained from the above estimated results.

Sample: 1971 2000		Included observations: 30			
Autocorrelation (AC)	Partial Correlation(PAC)	AC	PAC	Q-Stat	Prob
.|******* |	.|******* |	1 0.901	0.901	372.52	0.000
.|******* |	.|** |	2 0.862	0.269	714.52	0.000

(i) Do the residuals appear to be white noise series at 5 % level of significance? Justify your answer. **[2 Marks]**

(ii) Based on your findings what do you conclude about the specification of the model? Explain. **[1 Mark]**

Question 6

(a) The demand for Thailand's export of food crops (X) is given by:

$$\text{Ln } X_t = \beta_1 + \beta_2 \text{ Ln } (WY)_t + \beta_3 \text{ Ln } (PX)_t + e_t$$

where:

WY = world income measured by income of Thailand's main trading partners;
PX = index of export price of crops; and
t = time period

(i) What is the economic interpretation of β_2 and β_3? **[2 Marks]**

(ii) If $\beta_3 < -1$, would you recommend that Thai producers lower the price of food crops if they wish to increase their export revenue? Explain. **[2 Marks]**

(iii) If the above equation is modelled for Thailand's exquisite silk, explain the expected sign and magnitude of β_2. **[1 Mark]**

(iv) The results of the Breusch –pagan test are displayed as follows:

Breusch –Pagan for Heteroskedasticity
H_0: Constant Variance
Chi2(1) = 3.39
Prob > chi2 = 0.0657

What does the results above imply for the regression coefficients, standard errors, hypothesis testing and confidence intervals? **[2 Marks]**

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(b) STATA like other econometric software packages produces automatically a number called "Pseudo R^{2}" and "Likelihood ratio test." How are these numbers calculated and what can they be used for? **[3 Marks]**

(c) An analyst has observations on 86 countries, in three time periods, 1960, 1970 and 1980, which he uses to explain each country's per capita GDP growth rate (G) in terms of the explanatory variables:

pop = population growth
inv = share of output allocated to investment
igdp = initial level of GDP in 1960 in real terms
sec = human capital measured as the enrolment rate in secondary schools

The analyst considers three cross-sectional regressions, one for each of the years 1960, 1970 and 1980. The results from estimating a three equation with standard errors in parenthesis are provided below.

$$G_{60} = \underset{(0.0195)}{0.0231} - \underset{(0.2384)}{0.2435}\,POP_{60} + \underset{(0.0333)}{0.1280}\,INV_{60} - \underset{(0.0000020)}{0.0000021}\,IGDP_{60} + \underset{(0.0172)}{0.0410}\,SEC_{60}$$

$$G_{70} = \underset{(0.0313}{0.0185} - \underset{(0.4209)}{0.4336}\,POP_{70} + \underset{(0.0397)}{0.1870}\,INV_{70} - \underset{(0.0000018)}{0.0000026}\,IGDP_{70} + \underset{(0.0184)}{0.0127}\,SEC_{70}$$

$$G_{80} = \underset{(0.0265)}{0.0423} - \underset{(0.2997)}{0.8156}\,POP_{80} + \underset{(0.0297)}{0.1155}\,INV_{80} - \underset{(0.0000013)}{0.0000007}\,IGDP_{80} + \underset{(0.0141)}{0.0028}\,SEC_{80}$$

(i) What does the negative coefficient on POP suggest? Explain. **[2 Marks]**

(ii) Does human capital appear to influence growth rate? Justify your answer. **[2 Marks]**

(iii) Explain if the negative coefficient on IGDP is a reasonable outcome. **[1 Mark]**