# AFRICAN ECONOMIC RESEARCH CONSORTIUM <br> Collaborative PhD Programme in Economics for Sub-Saharan Africa <br> COMPREHENSIVE EXAMINATIONS IN CORE AND ELECTIVE FIELDS <br> FEBRUARY 14 - MARCH 6, 2018 <br> MICROECONOMICS 

Time: 08:00-11:00 GMT
Date: Monday, February 19, 2018

## INSTRUCTIONS:

Answer a total of FOUR questions: ONE question from Section A, ONE question from Section B, and TWO questions from Section C. Please note that Question 5 in Section C is compulsory.

The sections are weighted as indicated on the paper.

## SECTION A: (15\%) <br> Answer only ONE Question from this Section

## Question 1

(a) Discuss the cross price elasticity of demand and what its sign implies in terms of the relationship between goods.
[7.5 marks]
(b) Explain why the supply of houses is expected to be more price-inelastic in the short-run than in the long-run.
[7.5 marks]

## Question 2

(a) How does an externality give rise to market failure?
(b) What measures might be undertaken to correct such a market failure?
(c) Use one of the measures to illustrate how externalities can be corrected.

# SECTION B: (25\%) 

## Answer only ONE Question from this Section

## Question 3

(a) Derive the expenditure function for a perfect substitute utility function.
(b) Show that the derived expenditure function satisfies homogeneity property and Shephard's Lemma.
(c) Using your knowledge of Duality and Shephard's Lemma, derive the Slutsky equation.
[10 marks]

## Question 4

Given a production function $q=L^{1 / 2}+K^{1 / 2}$ where L and K are amounts of labour and capital used in producing q .
(a) Derive the elasticity of substitution for the production function.
(b) Determine the returns to scale of the production function.

Suppose the production function is specified as $q=L^{2} K^{2}$.
(c) Determine the elasticity of substitution of the production function.
(d) Determine the returns to scale of the production function.
(e) Compare the results in (b) and (d).

## SECTION C: (60\%)

## Answer TWO Questions from this Section,

## One of which MUST be Question 5, which is COMPULSORY

## Question 5 (Compulsory)

For each of the following statements define the underlined terms, and then determine whether the statement is true, false or uncertain with a brief explanation of your answer.
Answer any five
(a) All homothetic functions are homogeneous
(b) The diagonal terms of the substitution matrix for a demand system for two goods are always negative whereas the cross-price effects are not always symmetric.
[6 Marks]
(c) The Lagrangian multiplier in the cost minimization problem is the firm's marginal cost.
[6 Marks]
(d) The profit-maximization and cost-minimization approaches give same result for given output level.
[6 Marks]
(e) Every Walrasian Equilibrium allocation is Pareto efficient.
[6 Marks]
(f) The following Bernoulli utility function $u(x)=\sqrt{x}$ exhibits decreasing absolute risk aversion.
[6 Marks]
(g) A Bayesian game is a dynamic game with complete information.
[6 Marks]
(h) Unlike prospect theory gains and losses are given different weights in rational choice theory.
[6 Marks]

## Question 6

A couple (Husband, H and Wife, W) has one small car with a fuel tank capacity of 40 liters which cost $\$ 4$. The car does not get damaged except through non-use but it uses a minimum of 10 liter on any journey; thereafter, it uses fuel in multiples of 10 liters with a maximum permissible of 30 liters. At the starting point of the game the tank is full. Subsequently, the rules of the game are as follows:

- The wife makes the first move
- Each person must use the car when it is his/her turn
- The person that exhausts the fuel pays the other $\$ 4$ which could be used to refill the tank or spent in any other way he/she deems fit.
(a) Represent the information in a game tree making sure you explain the players' strategies and sequence of play.
[8 Marks]
(b) Characterize this game and explain the solution method that is appropriate.
[4 Marks]
(c) Identify and explain all possible Nash equilibria of the game.
[8 Marks]
(d) Find the subgame perfect Nash equilibrium if any and explain.
[10 Marks]


## Question 7

There are a large number of sellers and buyers in a used-car market. Each seller has one car to sell. Suppose the quality of a used car can be indexed by $\theta$, which is uniformly distributed on $[0,1]$. If a seller of type $\theta$ sells his car for a price $p$, he receives utility $u_{s}(\theta, p)$. A buyer who buys a car of quality at price $p$ receives a utility of $u_{b}=\theta-p$. If a seller does not sell his car, he receives a utility of zero. Similarly, a buyer who does not buy a car receives a utility of zero. A seller knows the quality of his car, but buyers only know the probability distribution of the quality of the car.
(a) Show that in a competitive equilibrium under asymmetric information $\mathrm{E}(\theta \mid \mathrm{p})=\mathrm{p}$.
(b) Show that if $u_{s}(\theta, p)=p-\theta / 2$, every $p \in(0,1 / 2]$ is an equilibrium price.
(c) Find the equilibrium price when $u_{s}(\theta, p)=p-\theta^{1 / 2}$ and what quality of cars are traded in equilibrium?
[6 marks]
(d) Find an equilibrium price when $u_{s}(\theta, p)=p-\theta^{3}$ and what quality of cars are traded in equilibrium?
[6 marks]
(e) Are any of the preceding outcomes Pareto efficient? Explain
[6 marks]

## Question 8

The inverse demand functions for the firms, firm 1 and firm 2, in a duopoly market are given as

$$
\begin{aligned}
& P_{1}=a_{1}-b_{1} y_{1}-c y_{2} \\
& P_{2}=a_{2}-c y_{1}-b_{2} y_{2}
\end{aligned} \quad \text { and }
$$

where $y_{1}$ and $y_{2}$ are the outputs of the firms and $P_{1}$ and $P_{2}$ are their prices respectively.
(a) Derive the direct demand functions and provide an index of product differentiation
[6 marks]
(b) Use the index to establish the conditions for the products of the firms to be considered (1) Perfect substitutes (2) Independent

Suppose that marginal costs are zero
(c) State the objective function of the firm 1 and derive the equilibrium output if the firms are Cournot competitors
(d) State the objective function of firm 1 and derive the equilibrium price if the firms are Bertrand competitors
[5 marks]
(e) Derive the reaction functions of firm 1 for both type of competition and comment on their slopes.
[6 marks]
(f) In each case, state whether the strategic variables are complements or substitutes

