AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative PhD Programme in Economics for Sub-Saharan Africa

COMPREHENSIVE EXAMINATIONS IN CORE AND ELECTIVE FIELDS

JANUARY 28 – FEBRUARY 17, 2020

ECONOMETRICS

Time: 08:00 – 11:00 GMT **Date: Friday, February 7, 2020**

INSTRUCTIONS:

1. Answer a total of FOUR questions: ONE question from Section A; ONE question from Section B; and TWO questions from Section C, One of which **MUST be either Question 5 or Question 6**.
2. The sections are weighted as indicated on the paper.
3. The null hypothesis and the alternative hypothesis for all the statistical tests in this examination should be indicated.

SECTION A: (15%)

Answer only ONE Question from this Section

Question 1

Consider the following simple linear regression equation:

$$y_i = \beta_0 + \beta_1 x_i + u_i \tag{1.1}$$

(a) Describe the specific principle and important assumptions of the Ordinary Least Squares (OLS) estimation method. **(5 Marks)**

(b) Why do we include an error term u in the regression equation? **(2 Marks)**

(c) The relationship between housing market price (*price*) and assessed housing value (*asprice*) was estimated using 108 observations. The results are as follows (standard errors are in parentheses):

$$\hat{price} = \underset{(16.24)}{-14.44} + \underset{(0.051)}{0.976}\ asprice \tag{1.2}$$

$R^2 = 0.762$ SSR=165.8

Use $\alpha = 0.05$ in all hypotheses testing.

(i) Interpret the regression results. **(5 Marks)**

(ii) Test the hypothesis of rational valuation, which states that a one unit change in *asprice* should be associated with a one unit change in *price*; that is, $H_0 : \beta_1 = 1$. **(3 Marks)**

Question 2

An output (y) variable was regressed on labor (L), capital (K) and raw materials (M) using observations on 35 firms. The OLS estimates and related quantities are as follows (standard errors are in parentheses):

$$\underset{(s.e.)}{\hat{y}_i} = \underset{(2.443)}{0.550} + \underset{(1.339)}{1.636} L_i + \underset{(1.160)}{1.661} K_i + \underset{(0.706)}{1.138} M_i \qquad (2.1)$$

$R^2 = 0.94$, $\hat{\sigma}^2 = 14.77$, $\text{RSS} = 265.78$, Breusch-Pagan test: $\text{BP} = 1.883$, $\text{pvalue} = 0.458$.

Correlation Matrix

	L	*K*	*M*
L	1.000		
K	0.952	1.000	
M	0.967	0.957	1.000

(a) Test for the significance of slope coefficients at 5% level of significance. **(3 Marks)**

(b) What is multicollinearity? Are there reasons to suggest that multicollinearity is a problem in the regression above? Explain. **(3 Marks)**

(c) Recommend ways to overcome the problem of multicollinearity. **(3 Marks)**

(d) What is heteroscedasticity? Is heteroscedasticity a problem in the regression above? Explain. **(3 Marks)**

(e) What are the consequences of heteroscedasticity in the context of OLS estimation? **(3 Marks)**

SECTION B: (25%)

<u>Answer only ONE Question from this Section</u>

Question 3

Refer to the plot of US federal funds rate (*fedrate*) and bond rate (*bondrate*) using quarterly time series observations from 1984q1 to 2009q4 in Figure 3.1.

Figure 3.1. Plot of Fedrate and Bondrate

(a) Comment on the behaviour of the two time series in Figure 3.1. **(3 Marks)**

(b) Show how Dickey-Fuller unit root test is derived using the random walk hypothesis. **(4 Marks)**

(c) Show that a random walk process is self-driven and has a long memory. **(5 Marks)**

(d) Based on results of the unit root tests below, determine the level of integration of each series. **(6 Marks)**

(e) What is cointegration? How do you test for cointegration of the two series using Engle-Granger method? **(4 Marks)**

(f) Based on the results provided below, test whether the two series are cointegrated. **(3 Marks)**

Null Hypothesis: FEDRATE has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.004819	0.1467
Test critical values:	1% level	-4.050509	
	5% level	-3.454471	
	10% level	-3.152909	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(FEDRATE) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic
Augmented Dickey-Fuller test statistic		-5.580625
Test critical values:	1% level	-4.050509
	5% level	-3.454471
	10% level	-3.152909

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: BONDRATE has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.077282	0.1592
Test critical values:	1% level	-4.050509	
	5% level	-3.454471	
	10% level	-3.152909	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(BONDRATE) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic
Augmented Dickey-Fuller test statistic		-7.897654
Test critical values:	1% level	-4.050509
	5% level	-3.454471
	10% level	-3.152909

Null Hypothesis: RESID has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.950651	0.0001
Test critical values:	1% level	-2.587831	
	5% level	-1.944006	
	10% level	-1.614656	

*MacKinnon (1996) one-sided p-values.

Question 4

(a) Consider the following model, $y_i = x_i'\beta + u_i$ for $i = 1, \ldots, n$, where the dependent variable is binary. Explain why the linear probability model (LPM) is not an appropriate model of choice. **(4 Marks)**

(b) Consider again the model in (a) with $y_i = 1$ if $y_i^* = x_i'\beta + \varepsilon_i > \alpha$ where α is nonnegative and x_i is a row vector of *k* columns. The zero-mean error term ε_i has a symmetric distribution.

(i) For which binary choice model is the variance of the error terms equal to $\frac{\pi^2}{3}$? **(1 Mark)**

(ii) Derive the log-likelihood function for the model in b(i). **(7 Marks)**

(iii) Show that the log-likelihood function is globally concave. **(5 Marks)**

(c) A researcher is interested in explaining what factors determine a family owning a house. He collected information from a sample of 40 families for each of the following variables:

$y_i = 1$ if a family owns a house; 0, otherwise.

x_{1i} = income in thousand US dollars.

x_{2i} = level of education: 1 = no education, 2 = primary, 3 = high school, 4 = university, and 5 = postgraduate.

Using these observations, the researcher estimated a binary Probit model and obtained the following results:

```
Probit regression                               Number of obs     =         40
                                                LR chi2(2)        =      13.99
                                                Prob > chi2       =     0.0009
Log likelihood = -20.532289                     Pseudo R2         =     0.2541
------------------------------------------------------------------------------
       house|      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .1949345   .0963663     2.02   0.043     .0060601    .383809
        educ |  -.0231153   .3438534    -0.07   0.946    -.6970557    .650825
       _cons |  -2.556841   .8001712    -3.20   0.001    -4.125148   -.988534
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .0566532   .0237133     2.39   0.017     .0101761   .1031304
        educ |  -.0067179   .0998887    -0.07   0.946    -.2024962   .1890603
------------------------------------------------------------------------------
```

(i) Interpret the estimated marginal effects. **(4 Marks)**

(ii) Comment on the overall significance and fitness of the model. **(4 Marks)**

SECTION C: (60%)

Answer TWO Questions from this Section,

ONE from Questions 5 and 6 ; and the OTHER from Questions 7 and 8

Question 5

(a) Given a 2-dimentional vector autoregressive model (where x_t and y_t are assumed stationary):

$$\begin{aligned} x_t &= c_1 + \alpha_{11}x_{t-1} + \alpha_{12}x_{t-2} + \cdots + \alpha_{1p}x_{t-p} + \beta_{11}y_{t-1} + \beta_{12}y_{t-2} + \cdots + \beta_{1p}y_{t-p} + u_{1t}; \\ y_t &= c_2 + \alpha_{21}x_{t-1} + \alpha_{22}x_{t-2} + \cdots + \alpha_{2p}x_{t-p} + \beta_{21}y_{t-1} + \beta_{22}y_{t-2} + \cdots + \beta_{2p}y_{t-p} + u_{2t} \end{aligned} \quad (5.1)$$

(i) Briefly explain the criteria you would use in determining the number of lags to include in the VAR model (5.1)? **(4 Marks)**

(ii) Suggest an estimation method for model (5.1)? Justify. **(3 Marks)**

(iii) What is Granger causality? **(2 Marks)**

(iv) Based on model (5.1), formulate hypotheses and indicate the test statistics to determine whether (1) y_t Granger causes x_t; and (2) x_t Granger causes y_t. **(6 Marks)**

(b) Consider the South Africa Quarterly GDP growth series: 1980Q1-2016Q2. Below are the plots of autocorrelation function (ACF) and partial autocorrelation function (PACF) computed up to 20 lags, and two tentative ARMA models that were identified and estimated using the series.

(i) Comment on the ACF and PACF. **(3 Marks)**

(ii) Analyse the statistics associated with the estimated AR (1) and MA (3) models and choose the most appropriate model for the series. **(5 Marks)**

(iii) Derive the mean and the variance of the model of your choice in (b)ii using the estimated results. **(3 Marks)**

(iv) Based on the model of your choice in (b)ii, estimate the one-step ahead forecast and the forecast variance if GDP 2016q2 is 65 million USD. **(4 Marks)**

Sample: 1 146
Included observations: 146

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.537	0.537	43.023	0.000
		2	0.345	0.079	60.905	0.000
		3	0.124	-0.127	63.227	0.000
		4	0.030	-0.019	63.366	0.000
		5	-0.040	-0.032	63.611	0.000
		6	-0.006	0.060	63.617	0.000
		7	-0.091	-0.128	64.913	0.000
		8	-0.072	0.005	65.721	0.000
		9	0.007	0.121	65.729	0.000
		10	0.047	0.015	66.081	0.000
		11	0.018	-0.072	66.130	0.000
		12	0.069	0.077	66.906	0.000
		13	0.023	-0.022	66.991	0.000
		14	-0.054	-0.118	67.467	0.000
		15	0.011	0.109	67.486	0.000
		16	-0.050	-0.074	67.906	0.000
		17	-0.034	0.021	68.101	0.000
		18	-0.031	-0.014	68.265	0.000
		19	-0.044	-0.045	68.597	0.000
		20	-0.035	0.047	68.811	0.000

AR (1) – Estimated Results

$$\underset{(s.e.)}{\hat{GDP}_t} = \underset{(0.0691)}{0.254} + \underset{(0.0693)}{0.538}\, GDP_{t-1} \qquad \hat{\sigma}^2 = 2.20$$

Figures in parentheses above are standard errors.

AR (1) Diagnostic Check

	Estimates	Q-Statistics	AIC and SIC
AR (1)	ϕ: 0.538 (0.0693)	Q (4): 2.927 (0.570)	AIC: 2.090
		Q (8): 8.215 (0.413)	SIC: 2.131
		Q (12): 11.219 (0.510)	

Note: Ljung-Box Q-statistics of the residuals from the fitted model. The p-values are in parentheses.

MA (3) – Estimated Results

$$\underset{(s.e.)}{\hat{GDP}_t} = \underset{(0.1205)}{0.570} + \underset{(0.0814)}{0.497}\, \varepsilon_{t-1} + \underset{(0.0833)}{0.441}\, \varepsilon_{t-2} + \underset{(0.0807)}{0.205}\, \varepsilon_{t-3} \quad \hat{\sigma}^2 = 2.46$$

Figures in parentheses are standard errors.

MA (3) Diagnostic Check

	Estimates	Q-Statistics	AIC and SIC
MA(3)	ϕ_1 : 0.497 (0.0814)	Q (4): 0.791 (0.374)	AIC: 2.097
	ϕ_2 : 0.441 (0.0833)	Q (8): 4.960 (0.421)	SIC: 2.179
	ϕ_3 : 0.205 (0.0807)	Q (12): 8.085 (0.526)	

Note: Ljung-Box Q-statistics of the residuals from the fitted model. The p-values are in parentheses.

Question 6

Consider the AR (1) process:

$$y_t = \phi y_{t-1} + \varepsilon_t \qquad (6.1)$$

where $\varepsilon_t \sim iid\left(0, \sigma^2\right)$ and $|\phi| < 1$.

(a) What is the OLS estimator of ϕ in (6.1)? **(2 Marks)**

(b) Derive the mean and the variance of (6.1). **(5 Marks)**

(c) The model in (6.1) is a special case of regression model that can be estimated using OLS where according to the *Greenberg and Webster* Central Limit Theorem (CLT), $\hat{\beta}$ is asymptotically normal, that is:

$$\hat{\beta}_T \approx N\left[\beta, \sigma^2 Q^{-1}/T\right].$$

Show that $\sqrt{T}\left(\hat{\phi}_T - \phi\right) \xrightarrow{d} N\left[0, \left(1-\phi^2\right)\right]$ **(6 Marks)**

(d) Explain why for $\phi = 1$ the usual t-statistic is not appropriate in hypothesis testing. **(3 Marks)**

(e) Assuming that y_t is a random walk process and ε_t is a Gaussian process, show that

$$y_t \sim N(0, \sigma^2 t)$$ **(4 Marks)**

(f) Assuming that (e) holds, a nondegenerate asymptotic distribution for $\hat{\phi}_T$ would be preferred for hypothesis testing. Show that

$$T\left(\hat{\phi}_T - 1\right) \xrightarrow{d} \frac{(1/2)\left\{\left[W(1)\right]^2 - 1\right\}}{\int_0^1 \left[W(r)\right]^2 dr}$$

where $\left[W(1)\right]^2$ is a $\chi^2(1)$ variable (chi-square distribution) and

$$T^{-2}\sum_{t=1}^{T} y_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 \left[W(r)\right]^2 dr$$ **(10 Marks)**

Question 7

(a) State three advantages and three limitations of panel data in modelling. **(6 Marks)**

(b) Consider the following panel data model: $y_{it} = \mu + x_{it}'\beta + \varepsilon_{it}$ where the variables and parameters are conventionally defined. The error term $\varepsilon_{it} = \alpha_i + \nu_{it}$, $i = 1,\ldots,N$ and $t = 1,\ldots,T$. What assumptions would you make to efficiently estimate the model using a random effect approach? **(6 Marks)**

(c) Show that the variance-covariance matrix of the error vector ε in (b) is given by: $\Omega = I_N \otimes A$ where A is a $T \times T$ matrix. **(10 Marks)**

(d) Show that

$$A^{-1/2} = \frac{1}{\sigma_\nu}\left\{ I_T - \left[\left(\frac{1-\phi}{T} \right) \iota_T \iota_T' \right] \right\} \text{ with } \phi = \sqrt{\frac{\sigma_\nu^2}{T\sigma_\alpha^2 + \sigma_\nu^2}}$$

(8 Marks)

Question 8

(a) A fertility analysis is conducted using a discrete choice model. It aims at assessing the determinants of the binary variable denoted by ENF, taking on the values 1 (at least one child) and 0 (no child).

Some socio-economic characteristics assumed to affect ENF are defined as follows:

- DIPL=Education: 1 if the head of household did not complete high school education, 2 if the head of household completed high school education, and 3 if the head of household has a university education;
- AGE= age of household head in years; and
- AGE2 = age squared divided by 100.

The STATA output provided below used the robust variance estimation. The sample contains adults of ages between 20 to 60. The variables $\_Idipl\_1$, $\_Idipl\_2$, and $\_Idipl\_3$ are dummies derived from *DIPL*.

```
. xi: probit enf i.dipl age age2
i.dipl            _Idipl_1-3          (naturally coded; _Idipl_1 omitted)

Iteration 0:   log likelihood = -18636.845
Iteration 1:   log likelihood = -17371.858
Iteration 2:   log likelihood = -17369.568
Iteration 3:   log likelihood = -17369.568

Probit estimates                                  Number of obs   =      28922
                                                  LR chi2(4)      =    2534.55
                                                  Prob > chi2     =     0.0000
Log likelihood = -17369.568                       Pseudo R2       =     0.0680

------------------------------------------------------------------------------
         enf |      Coef.   Std. Err.       z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    _Idipl_2 |  -.1324138   .0236364    -5.60   0.000    -.1787404   -.0860873
    _Idipl_3 |  -.2885005    .018667   -15.46   0.000     -.325087   -.2519139
         age |   .2996346    .006378    46.98   0.000     .2871339    .3121352
        age2 |  -.3834785   .0080627   -47.56   0.000    -.3992812   -.3676758
       _cons |  -4.966942   .1199125   -41.42   0.000    -5.201966   -4.731918
------------------------------------------------------------------------------
. vce

             | _Idipl_2 _Idipl_3      age     age2    _cons
-------------+---------------------------------------------
    _Idipl_2 |  .000559
    _Idipl_3 |  .000104  .000348
         age |  5.3e-06 -4.8e-06  .000041
        age2 | -3.7e-06  7.8e-06 -.000051  .000065
       _cons | -.000248 -.000041 -.000755  .000932  .014379
```

(i) Based on the model results above, how do you obtain the regression coefficients for the other competing nonlinear model? Derive the factor used by Amemiya for this approximation. **(5 Marks)**

(ii) Interpret the education coefficients. **(4 Marks)**

(iii) Test the hypothesis that the coefficients of $\_Idipl\_2$ and $\_Idipl\_3$ are equal. **(5 Marks)**

(b) Using the same sample in (a) the researcher estimated an ordered probit model. The dependent variable NENF takes on values 0, 1, 2, 3 or 4, if the number of children in the household is 0, 1, 2, 3, or at least 4, respectively. The ordered probit model is then estimated with unknown cut-offs s_l for $l = 0,1,K,5$:

$NENF_i = j \in \{0,1,2,3,4\}$ if $s_j < x_i'\beta + u_i \le s_{j+1}$ with $u_i : N(0,\sigma^2)$. It is assumed that

$s_0 = 0$ and $s_5 = +\infty$.

The STATA output provided below used the robust variance estimation (the parameters $\_cut1$, …, $\_cut4$ denote the thresholds or cut-offs s_1, …, s_4 of the model, respectively).

```
. xi: oprobit nenf i.dipl age age2
i.dipl            _Idipl_1-3          (naturally coded; _Idipl_1 omitted)

Iteration 0:   log likelihood = -40356.058
Iteration 1:   log likelihood = -39014.837
Iteration 2:   log likelihood = -39013.178

Ordered probit estimates                          Number of obs   =      28922
                                                  LR chi2(4)      =    2685.76
                                                  Prob > chi2     =     0.0000
Log likelihood = -39013.178                       Pseudo R2       =     0.0333

------------------------------------------------------------------------------
        nenf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    _Idipl_2 |  -.1145041   .0194437    -5.89   0.000    -.1526129   -.0763952
    _Idipl_3 |  -.2033115   .0155256   -13.10   0.000    -.2337412   -.1728817
         age |     .26945   .0054888    49.09   0.000     .2586921     .280208
        age2 |  -.3453101    .006959   -49.62   0.000    -.3589496   -.3316706
-------------+----------------------------------------------------------------
       _cut1 |   4.436445   .1032947          (Ancillary parameters)
       _cut2 |   5.178941   .1041345
       _cut3 |   6.082835   .1050567
       _cut4 |    6.83505   .1058442
------------------------------------------------------------------------------

. vce

             | _Idipl_2 _Idipl_3      age     age2    _cut1    _cut2    _cut3
-------------+---------------------------------------------------------------
    _Idipl_2 |  .000378
    _Idipl_3 |  .000066  .000241
         age |  3.9e-06 -1.9e-06   .00003
        age2 | -3.1e-06  3.3e-06 -.000038  .000048
       _cut1 |  .000168  .000047  .000559 -.000692   .01067
       _cut2 |  .000167  .000043  .000564 -.000698  .010729  .010844
       _cut3 |  .000165  .000041  .000569 -.000704  .010794  .010898  .011037
       _cut4 |  .000162  .000039  .000569 -.000705  .010803  .010903  .011025

             |    _cut4
-------------+---------
       _cut4 |  .011203
```

(i) Show that the binary probit model is a special case of the ordered probit by computing $\Pr\left(ENF_i = 1 | x_i\right)$ as a function of the threshold s_1. What can you conclude about the intercept of the binary probit model in relation to s_1? **(5 Marks)**

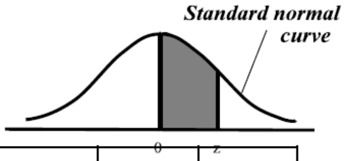
(ii) Calculate the probability of having 3 children for a household head who did not complete high school education and is 35 years old. Hint: A table of the standard normal distributed is provided in Appendix 1. **(5 Marks)**

(iii) The estimated marginal effects associated with those household heads who completed high school education are given below:

	NENF	dy/dx
	0	0.1003
	1	0.3252
Idipl_2	2	0.2002
	3	-0.3096
	4	-0.3161

Interpret the results. **(6 Marks)**

Table 1: Areas under the Standard Normal Curve



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0	0	0.01	0.012	0.016	0.02	0.024	0.028	0.032	0.036
0.1	0.04	0.044	0.048	0.052	0.056	0.06	0.064	0.068	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
l.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.483	0.4834	0.4838	0.4842	0.4846	0.485	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.489
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.492	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.494	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.496	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.497	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.498	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.499	0.499

Table 2: The t Distribution

The entries in the table give the critical values of t for the specified number of degrees of freedom and areas in the right tail.

d.f.(v)	Areas in the right tail under the t distribution curve (α)					
d.f.(v)	0.100	0.050	0.025	0.010	0.005	d.f.(v)
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
inf	1.282	1.645	1.960	2.326	2.576	inf

Critical Values of t