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AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative Masters Programme in Economics for Anglophone Africa (Except Nigeria)

JOINT FACILITY FOR ELECTIVES (JFE) 2018
JUNE – SEPTEMBER

ECONOMETRICS THEORY AND PRACTICE II

Second Semester: Final Examination

Duration: 3 Hours **Date: Friday, September 21, 2018**

INSTRUCTIONS:

1. This examination is divided into two parts: **Section A** and **Section B**.
2. You are required to answer **ANY TWO QUESTIONS** in **EACH SECTION**.
3. All questions carry 15 marks each.

Section A:

Answer ANY TWO Questions in this Section

Question 1

The following panel data model is considered:

$$y_{it} = \delta + z'_{it}\beta + u_{it},\ i = 1,\ldots,N;\ t = 1,\ldots,T \quad (1).$$

with $$u_{it} = \mu_i + \nu_{it} \quad (2)$$

where i denotes the cross-section, t the time period and z_{it} the set of K explanatory variables corresponding to individual i at period t. In addition, δ is a scalar and β a $K \times 1$ vector of slope coefficients. In Equation (2), μ_i accounts for the unobservable individual-specific effect whereas ν_{it} stands for the remainder disturbance, which is assumed spherical.

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(i) From a general perspective, provide two advantages and two limitations of the use of panel data in empirical economic research. **[2 marks]**

(ii) One considers the random effect approach, assuming individual specificities to behave as error terms. Define the required assumptions about the error components μ_i and v_{it}, at the observation level and then in matrix form. **[3 marks]**

(iii) Would you consider the overall error term u_{it} as spherical? In order to provide a justified answer, show the relation below and then conclude:

$$\operatorname{cov}\left(u_{it}, u_{js}\right) = \begin{cases} \sigma_\mu^2 + \sigma_v^2 & \text{if } i = j \text{ and } t = s \\ \sigma_\mu^2 & \text{if } i = j \text{ and } t \neq s \\ 0 & \text{if } i \neq j \end{cases}. \qquad (3)$$

[2 marks]

(iv) Our researcher is not convinced by the well-known Hausman test procedure which has led to the rejection of the null hypothesis. He would like to know more about the Chamberlain restrictions.

(a) Based on the Chamberlain approach, show that the individual effect μ_i can be written as

$$\mu_i = z_i'\lambda + \varepsilon_i \quad \forall i = 1, \ldots, N \qquad (4)$$

with all the components involved clearly defined. **[2 marks]**

(b) After setting $\delta = 0$ in Equation (1), show that the dependent variable y_{it} can be rewritten as: $\forall i = 1, \ldots, N$ and $t = 1, \ldots, T$,

$$y_{it} = z_{it}'\left(\beta + \lambda_t\right) + \sum_{s \neq t} z_{is}'\lambda_s + \eta_{it} \qquad (5)$$

[2 marks]

(c) Deduce from Equation (5) the reduced form below (all matrices and vectors involved should be clearly stated accordingly):

$$y_i' = z_i'\pi + \eta_i, \qquad \forall i = 1, \ldots, N \qquad (6)$$

[4 marks]

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Question 2

You have just been appointed as a consultant by a great international organization. You want to make an inaugural speech about the estimation challenges raised by the binary response models.

(i) After providing an economic example of a binary response model, present the econometric motivation of your example by means of the random utility approach. Hint: *You will have to express the probability of success as a function a certain unspecified distribution with probability density function denoted by g.* **[3.5 marks]**

(ii) A participant in the audience argues that the Linear Probability Model (LPM, hereafter) is a valid estimation method because of its simplicity, efficiency and correct prediction capability. Explain why you disagree by revealing the true caveats of this linear technique. **[1.5 marks]**

(iii) Another major concern that has been raised is the concavity of the log-likelihood function, which is seen as too complex to prove, even in the ideal case of linearity. Your aim is therefore to address this question assuming the use of the maximum likelihood approach for the LPM.

(a) Establish the likelihood and the corresponding log-likelihood functions of the general model (including the above undefined function g). Then apply it to the LPM case. **[4 marks]**

(b) Derive the first order condition of the log-likelihood maximization under this general specification (using the function g), and then apply it again to the LPM case. **[3 marks]**

(c) Deduce the second order condition of the log-likelihood maximization in the case of the LPM and show that the Hessian matrix is negative definite. **[3 marks]**

Question 3

We consider a population in which a fraction α_1 of this population has a constant hazard equal to h_1 whereas the other proportion $\alpha_2 = 1 - \alpha_1$ of the population has a greater constant hazard equal to $h_2 > h_1$. The density of the related duration variable T is given by:

$$f(t) = \alpha_1 h_1 \exp(-h_1 t) + \alpha_2 h_2 \exp(-h_2 t).$$

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(i) Derive the cumulative density function $F(t)$ of the duration time T. **[3 marks]**

(ii) Deduce the survival function $S(t)$. **[1 mark]**

(iii) What is the hazard function of T? Under which condition is the temporal independence still holding? **[4 marks]**

(iv) What is the mean duration time $E(T)$? **[2 marks]**

(v) Show, by integration by parts, that $\int_0^{+\infty} t^2 h \exp(-ht)\,dt = \frac{2}{h^2}$. Deduce the variance of the duration time T. **[5 marks]**

Section B:

Answer ANY TWO Questions in this Section

Question 4

Let (y_i, x_i, d_i) for $i = 1, 2, \ldots, N$ be the vector of observations on a scalar-valued outcome variable y, a vector of observable explanatory variables x, and a binary indicator of a treatment variable d. The outcome variable of the treated individual is denoted by y_{1i} whereas the one for the control group is y_{0i}.

(i) Present three different versions of the conditional independence assumption and explain each of them. **[3 marks]**

(ii) Define the propensity score $p(x)$. **[1 mark]**

(iii) Prove the Rosenbaum and Rubin (1983) corollary which states that the conditional independence assumption implies that $y_0, y_1 \perp d \,|\, p(x)$. How can you interpret this result? **[5 marks]**

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(iv) We now express the response variable y as a linear combination of the group-outcomes:

$$y=(1-d)y_0+dy_1.$$

(a) Show that $\left[d-p(x)\right]y=\left[1-p(x)\right]dy_1-p(x)(1-d)y_0$ (Hint: do not forget that $d^2=d$ as $d\in\{0,1\}$) **[3 marks]**

(b) Deduce the following expression of the average treatment effect for a propensity matching score model:

$$ATE=E(y_1-y_0)=E\left\{\frac{\left[d-p(x)\right]y}{p(x)\left[1-p(x)\right]}\right\}$$ **[2 marks]**

(c) Propose a consistent estimator of the average treatment effect of question 4.b), based on a sample of size N. **[1 mark]**

Question 5

(i) What is the most popular one-parameter model used to deal with count data problems? Write down the related probability density function. Deduce and explain the equidispersion and heteroscedasticity properties encountered when using this model. **[4 marks]**

(ii) Derive the log likelihood function of the Poisson model with x_i $(i=1,2,\ldots,N)$ as a $K\times 1$ vector of regressors and β the corresponding vector of coefficients. **[4 marks]**

(iii) Deduce the first order conditions of the maximum likelihood estimation procedure. **[2 marks]**

(iv) Explain how the quasi or pseudo maximum likelihood differ from the usual maximum likelihood estimation. **[1 mark]**

(v) Present the Linear Exponential Family (LEF, hereafter) by providing:

(a) The expression of the probability density function; and **[1 mark]**

(b) Three standard distributions belonging to the LEF. **[3 marks]**

Question 6

An individual is faced with a unique choice to make between $J+1$ unordered alternatives, with J being at least equal to 2. The dependent variable (the choice) is made based on a set of k explanatory factors in a sample of size N.

(i) Cite at least two econometric models suitable to such context, in addition to the well-known multinomial logit model (MNL, hereafter). **[2 marks]**

(ii) We now consider the MNL. After explaining under which circumstances this model is appropriate, write down the probability p_{ij} for an individual i to choose a given option j knowing his/her characteristics x_i, where $j \in \{0,1,\ldots,J\}$. Deduce the value of the probability of choosing the base category $j=0$. **[4 marks]**

(iii) Show that $\frac{p_{ij}}{p_{i0}} = \exp\left(x\beta_j\right)$; $j=1,\ldots,J$. Deduce that $\ln\left(\frac{p_{ij}}{p_{ih}}\right) = x\left(\beta_j - \beta_h\right)$ for $j \neq h$. **[4 marks]**

(iv) Explain and write down the conditional likelihood and its logarithmic version of the MNL model. **[5 marks]**