AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative PhD Programme in Economics for Sub-Saharan Africa
COMPREHENSIVE EXAMINATIONS IN CORE AND ELECTIVE FIELDS
FEBRUARY 13 - MARCH 3, 2017

## MICROECONOMICS

Time: 08:00-11:00 GMT
Date: Friday, February 17, 2017
INSTRUCTIONS:

Answer a total of FOUR questions: ONE question from Section A, ONE question from Section B, and TWO questions from Section C. Please note that Question 5 in Section C is COMPULSORY.

The sections are weighted as indicated on the paper.

## Section A: (15\%)

Answer only ONE Question from this Section

## Question 1

Explain why demand curves slope down to the right for all normal goods and some inferior goods.
[15 Marks]

## Question 2

(a) Using a graph, explain the relation between average and marginal cost curves.
[7 Marks]
(b) In a competitive market in the short run, state and explain the condition under which a firm produces a positive output level.

## Section B: ( $\mathbf{2 5 \%}$ )

## Answer only ONE Ouestion from this Section

## Question 3

A production function for a perfectly competitive firm is given as:

$$
q=A x_{1}^{1 / 2} x_{2}^{1 / 2}
$$

where $q$ is the output (in tons) and $x_{i}$ are amounts of inputs used in the production process. The cost of producing $q$ is given by:

$$
C=\sum_{i} w_{i} x_{i}, \quad i=1,2
$$

where $w_{i}$ are per unit input prices for input 1 and input 2.
(a) Derive the cost function in its general form and verify that it satisfies the homogeneity property and Shephard's Lemma.
[10 Marks]
(b) When $w_{1}=w_{2}=100$ and $\mathrm{A}=10$, what is the minimum amount of each input required to produce 360 tons of output?
(c) Compute the optimal profit for the firm when $p=20$.
(d) Compare and contrast the Cobb Douglas and constant elasticity of substitution (CES) production functions.

## Question 4

Consider an economy with two consumers A and B . Consumers A and B have utility functions $u\left(x_{1}^{A}, x_{2}^{A}\right)=\left(x_{1}^{A}\right)^{1 / 4}\left(x_{2}^{A}\right)^{1 / 2}$ and $u\left(x_{1}^{B}, x_{2}^{B}\right)=\left(x_{1}^{B}\right)^{1 / 4}\left(x_{2}^{B}\right)^{1 / 2}$, respectively. They face prices $p_{1}$ and $p_{2}$ for good 1 and good 2 , respectively, and they have incomes $I^{A}$ and $I^{B}$, respectively.
(a) Write formally the economic problem faced by consumer $A$ and derive the Marshallian demand functions $x_{1}^{* A}\left(p_{1}, p_{2}, I^{A}\right)$ and $x_{2}^{* A}\left(p_{1}, p_{2}, I^{A}\right)$. [5 Marks]
(b) Explain Walras' Law. Why is this law important for the general equilibrium theory?
[8 Marks]
(c) If the consumers have initial endowments $e^{A}=(3,3)$ and $e^{B}=(9,6)$, compute the Walrasian equilibrium relative price, and allocations.

# Section C: ( $60 \%$ ) <br> Answer TWO Questions from this Section, One of which MUST be Question 5, which is COMPULSORY 

## Question 5 (Compulsory)

For each of the following statements define the underlined terms then determine whether the statement is True, False or Uncertain with a brief explanation of your answer. Answer any five of the sub-questions in this question.
(a) The Kuhn Tucker conditions are valid only for solving constrained optimization problems with boundary conditions.
(b) A risk averse agent will always prefer the expected value of a gamble for sure than the gamble itself.
(c) Hotelling's lemma allows us to derive the supply function from the profit function
(d) If a natural monopoly is to operate at the efficient output level by adopting marginal cost pricing, it will need a subsidy to stay in production.
(e) In the repeated prisoner's dilemma game the Folk theorem provides the necessary condition for the Pareto efficient outcome to be sustainable.
(f) Equivalent variation and Compensating Variation always give identical measurements of welfare changes.
(g) In the presence of negative externalities a firm will produce more than the socially optimal output.
(h) The lemon market of Akerlof illustrates a moral hazard problem.

## Question 6

Consider the following indirect utility function derived from a consumer's utility maximization:

$$
v(p, y)=\alpha_{1}\left(\frac{y}{p_{1}}\right)^{\beta_{1}}+\alpha_{2}\left(\frac{y}{p_{2}}\right)^{\beta_{2}}
$$

where $y$ is income, $p_{i}$ is price of commodity $i$, for $i=1,2$. and $\alpha_{i}$ and $\beta_{i}$ are parameters.
(a) Determine the Lagrange multiplier of the underlying consumer's utility maximization problem.
[8 Marks]
(b) Determine the Marshallian demand functions.
(c) Explain the concept of duality in consumer theory.

## Question 7

Two friends, Juma and Amina, are considering a time out together and need to take decisions simultaneously on what activity to engage in. Juma loves visiting ( $V$ ), Amina loves the theater $(T)$. The payoff to each from choosing either of the two activities given the choice of the other is represented in the game matrix below

| Juma Amina | $\boldsymbol{V}$ | $\boldsymbol{T}$ |
| :---: | :---: | :---: |
| $\boldsymbol{V}$ | 3,1 | 0,0 |
| $\boldsymbol{T}$ | 0,0 | 2,5 |

(a) Identify the pure strategy Nash equilibrium (equilibria) if any.
[3 Marks]
(b) Is there any Nash equilibrium in mixed strategies? If yes derive the mixed strategy Nash equilibrium.
[5 Marks]
(c) Draw the extensive form of the game and determine which of the pure strategy Nash equilibria is credible.
(d) Game theory has become a major tool of analysis in economics. Provide a critique of this approach.
[15 Marks]

## Question 8

Consider the following principal-agent situation. We have a principal P and an agent A . P wants to hire A for a one time project. If A works for P , A can choose high effort, $e_{H}$, or low effort, $\mathrm{e}_{\mathrm{L}}$. Profits are either high, $\pi_{H}$, or low $\pi_{\mathrm{L}}$, with $\pi_{\mathrm{H}}>\pi_{L}$. If A chooses $e_{H}$ then profits are $\pi_{H}$ with probability $\rho_{H}$, and $\pi_{L}$ with probability $1-\rho_{H}$. If A chooses $e_{L}$ then profits are $\pi_{H}$ with probability $\rho_{L}$ and $\pi_{L}$ with probability $1-\rho_{L}$. P maximizes expected profits from the project, less the expected wages paid to the agent, A maximizes expected utility as follows, given a wage, $w$, and effort choice, $e: U(w, e)=V(w)-e$ where $V^{\prime}(w)>0$ and $V^{\prime \prime}(w)<0$. P designs a contract, A then accepts it or not, and if A accepts, A then chooses an effort level. Assume that A has a reservation utility level of $U_{0}$.
(a) Show how to implement $e_{L}$ and $\mathrm{e}_{\mathrm{H}}$ if effort is observable and verifiable and the agent is risk averse.
[10 Marks]
(b) Answer part (a) above given that the agent is risk neutral.
(c) If $e$ is not observable, and the agent is risk-neutral, show that P can still obtain the same payoffs as in case (a).
[15 Marks]

