



## AFRICAN ECONOMIC RESEARCH CONSORTIUM

*Collaborative PhD Programme in Economics for Sub-Saharan Africa*

### **COMPREHENSIVE EXAMINATIONS IN CORE AND ELECTIVE FIELDS**

**FEBRUARY 13 – MARCH 3, 2017**

### **MACROECONOMICS**

**Time: 08:00 – 11:00 GMT**

**Date: Monday, February 13, 2017**

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#### **INSTRUCTIONS:**

Answer a total of FOUR questions: ONE question from Section A, ONE question from Section B, and TWO questions from Section C (at least one of which MUST BE Question 5 or 6).

The sections are weighted as indicated on the paper.

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#### **SECTION A: (15%)**

**Answer only ONE Question from this Section**

#### **Question 1**

Suppose that an economy is characterized by the following behavioural equations:

$$C = c_0 + c_1 Y_D$$

$$Y_D \equiv Y - T$$

$$I = \bar{I}$$

$$G = \bar{G}$$

$$T = \bar{T}$$

(a) Solve for:

(i) Equilibrium GDP ( $Y$ ). **[2 Marks]**

(ii) Interpret the term on the right-hand side of (a) when income is zero. **[2 Marks]**

(iii) Derive the investment multiplier. **[2 Marks]**



- (b) Suppose that taxes depend linearly on income, according to the equation  $T = T_0 + t_1 Y$ , where  $t_1$  is the tax rate, and is between 0 and 1.
- (i) Find the equation for equilibrium GDP. **[2 Marks]**
- (ii) Find the expression for the tax multiplier. Is the multiplier when taxes are endogenous, larger than, smaller than, or the same as when taxes are exogenous? **[3 Marks]**
- (c) Distinguish between changes in  $c_0$  and changes in  $c_1 Y_D$ . **[4 Marks]**

## Question 2

- (a) Why are cheques and credit cards not money? **[4 Marks]**
- (b) What are the defining features of classical macroeconomics and what policies do classical macroeconomics suggest with regard to unemployment and the role of government? **[6 Marks]**
- (c) Using the Quantity Theory of Money, show how the quantity of money triggers inflation in the economy. **[5 Marks]**

## SECTION B: (25%)

Answer only ONE Question from this Section

## Question 3

The household's utility in period  $t$  is derived from consumption and employment, which is summarized:

$$U_t = \log C_t + \psi \log (1 - N_t)$$

The budget constraint for this problem is then expressed as;

$$P_t C_t = W_t N_t$$

where  $C_t$ ,  $N_t$ ,  $P_t$  and  $W_t$  refer to consumption, employment, prices of consumer goods, and wages. It is then assumed that  $\psi > 0$ .

- (a) Derive the first order conditions for the household, with respect to consumption and employment. **[13 Marks]**
- (b) How does labour supply depend on the wage in this example? **[12 Marks]**



#### Question 4

Suppose an individual lives for 2 periods in an economy where money exists and is valued. Utility maximization problem for the individual born in time  $t$ ,  $t \geq 0$ , is expressed as:

$$\text{Max. } u(C_{1t}, C_{2t+1})$$

subject to

$$P_t(1 - C_{1t}) = M_t^d \text{ and } P_{t+1}C_{2t+1} = M_t^d,$$

where  $C_{it}$  and  $P_t$  denote consumption and price at time  $t$ , and  $M_t^d$  is the individual's demand for money.

- (a) Explain how this individual consumes in both periods. **[5 Marks]**
- (b) If there is no intrinsic uncertainty in this problem, and so perfect foresight is assumed, which means the actual and expected prices at time  $t + 1$  are the same. What is the first order condition for utility maximization of this individual? **[10 Marks]**
- (c) Write the implied demand for money function as of time  $t$ , and briefly explain its intuition. **[10 Marks]**



## SECTION C: (60%)

Answer TWO Questions from this Section,

AT LEAST one of which MUST BE Question 5 or 6

### Question 5

Consider the new Keynesian Phillips curve with indexation, which may be written as,

$$\pi_t = \frac{1}{1 + \beta} \pi_{t-1} + \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \chi y_t, \quad 1 > \chi > 0$$

where  $\pi_t$  represents inflation, derived from the price index ( $p_t$ ), and  $y_t$  is a measure of real output.  $m_t$  is a measure of nominal output and can be expressed as,  $m_t = y_t + p_t$ , which is the target of central bank monetary policy.

All of the above variables are expressed in terms of natural logarithms.

- (a) Under the assumptions of perfect foresight and  $\beta = 1$ , express  $p_t$  in terms of  $m_t$ ,  $p_{t+1}$ ,  $p_{t-1}$  and  $p_{t-2}$  **[8 Marks]**
- (b) If we assume that the values of  $p_{t+1}$ ,  $p_{t-1}$  and  $p_{t-2}$  remain constant, what would be the effect of an increase in  $m_t$  on current prices? **[7 Marks]**
- (c) If it is assumed that there is a positive relationship between  $m_t$  and  $p_{t+1}$ , would the effect of an increase in  $m_t$  on current prices, be greater or less than in the previous case? **[7 Marks]**
- (d) If we now assume that  $\chi = 0.5$ , would the use of the new Keynesian Phillips curve without indexation allow for a stronger or weaker relationship between  $m_t$  and  $p_t$ ? Consider the cases where there is, and is not, a relationship between  $m_t$  and  $p_{t+1}$  and the new Keynesian Phillips curve without indexation is written as,

$$\pi_t = \beta E_t \pi_{t+1} + \chi y_t$$

**[8 Marks]**

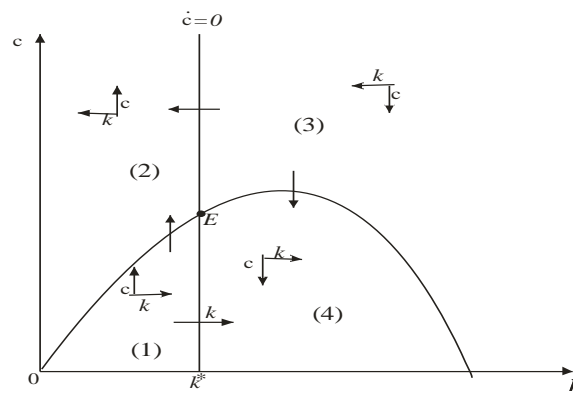


## Question 6

Consider a Ramsey-Cass-Koopmans type growth model in which the equations of motion of consumption per unit of effective labour and capital stock per unit of effective labour are

$$\dot{c}(t) = \frac{r(t) - \rho - \theta g}{\theta} c(t) \quad \text{and} \quad \dot{k}(t) = [f(k(t)) - c(t)] - [(n + g)k(t)], \quad \text{respectively.}$$

The phase diagram associated with the model is drawn below. The  $\dot{c} = 0$  line and the  $\dot{k} = 0$  locus divide the figure into quadrants 1, 2, 3 and 4. Use your knowledge of the workings of the model to answer the following questions. [ $c$  represents consumption per unit of effective labour,  $n$  is population growth,  $r$  is rental price of capital,  $k$  is capital per unit of effective labour,  $g$  denotes growth of knowledge and  $\rho$  is discount rate]



- Why is  $\dot{c} > 0$  and  $\dot{k} > 0$  in quadrant 1? [6 Marks]
- Explain the dynamics in quadrant 2. [6 Marks]
- Why is  $\dot{c} < 0$  and  $\dot{k} < 0$  in quadrant 3? [6 Marks]
- At point E, what happens to consumption per worker and output per worker? [2 Marks]
- Explain the effects of a decline the discount rate,  $\rho$  on the  $\dot{c} = 0$  line and the  $\dot{k} = 0$  curve. Explain the adjustment process. [10 Marks]



## Question 7

Consider a standard optimal growth model in continuous time in which the aggregate production function is given by:

$$Y(t) = F[k(t), N(t)],$$

where  $F(\cdot)$  has standard properties and  $N(t)$  is growing at the rate  $n > 0$ . The depreciation rate of capital is given by  $\delta > 0$ . The single household inelastically supplies labour each period and then chooses consumption and savings in order to maximize

$$\int_0^{\infty} e^{-\rho t} U(c(t)) N(t) dt,$$

where  $c(t) = \frac{c(t)}{N(t)}$  is per-capita consumption and  $U(\cdot)$  has the functional form:

$$U(c(t)) = \begin{cases} \frac{c(t)^{1-q}}{1-q}; q \neq 1 \\ \ln c(t); q = 1 \end{cases}$$

It is assumed that all parameter values are such that a well behaved equilibrium exists. In addition to output produced via production function, output arrives exogenously every period at the rate  $\bar{f}$  units per person. Given this environment do the following:

- Express the social planner problem in intensive form (i.e. per-capita) form – show your derivation. **[6 Marks]**
- Write down the Hamiltonian for this problem and derive the necessary conditions; including the transversality condition. **[8 Marks]**
- Derive the phase diagram for the economy (be sure to explain your diagram). **[8 Marks]**
- Define the steady-state. What fraction of the exogenous output,  $\bar{f}$  is consumed in the steady-state? Why? **[8 Marks]**



## Question 8

In the New Keynesian model of the business cycle it is assumed that the representative household will consume a variety of different goods that are indexed by  $j$ , which has a continuous distribution that may be formulated as  $j \in [0,1]$ . The broad consumption index,  $C_t$ , could then be expressed as

$$C_t = \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

where  $C_{j,t}$  is the quantity of good  $j$  consumed by the representative household and  $\epsilon$  is the marginal rate of substitution between goods. The budget constraint for this household is represented as,

$$\int_0^1 P_{j,t} C_{j,t} dj = W_t N_t,$$

where  $P_{j,t}$  refers to the price of good  $j$ ,  $W_t$  refers to wages and  $N_t$ , refers to labour.

- (a) Derive an expression that identifies the optimal value of  $C_{j,t}$  as a function of prices and the index of total consumption (i.e.,  $C_t$ ). **[16 Marks]**
- (b) Use this expression to derive the aggregate price index of all goods. **[14 Marks]**