Optimal Monetary Policy with Inflation, Output and Asset Price Volatility in an Open Economy

By

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Bringing Rigour and Evidence to Economic Policy Making in Africa

Optimal Monetary Policy with Inflation, Output and Asset Price Volatility in an Open Economy

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List of abbreviations and acronyms

- DSGE Dynamic Stochastic General Equilibrium
- GSE Ghana Stock Exchange
- GSE-CI Ghana Stock Exchange Composite Index
- NASI Nairobi All Share Index
- NSE Nairobi Securities Exchange

Abstract

This paper aims to establish optimal response of monetary policy to output, inflation, and asset price volatility in small open economies of Kenya and Ghana. The paper estimates a monetary policy response function for inflation, asset prices, and output volatility developed from a dynamic stochastic general equilibrium model using quarterly data from 2000 to 2018. The analysis shows that monetary policy accord inflation greatest weight compared to output and asset prices. However, there are differences in the sensitivity of monetary policy across the economies, and hence price, output, and welfare outcomes. The prioritization of inflation stifles output growth more in Ghana than in Kenya due to high interest rate. Despite monetary policy prioritizing inflation in Ghana, average inflation is higher compared to Kenya. Results from dynamic optimization shows that, a consistent intervention in the economy to stabilize inflation, output, nominal exchange rate, and asset prices, achieves higher welfare.

Key words: Optimal monetary policy; Output fluctuations; Prices; Volatility. *JEL classification codes:* E52; E44; E47; G11; G12.

1. Introduction

Monetary authorities in developing economies desire to hasten economic growth and maintain price stability simultaneously. Interest rate is employed to regulate prices, which in turn influence the level of utilization of the economy's resources and the size of output gap. However, on the one hand, elevated interest rate geared towards maintaining price stability stifles growth (Woodford, 2003; Divino, 2009). On the other hand, extended periods of stable inflation and robust growth precipitate asset price misalignment. Furthermore, a small open economy is exposed to external shocks that affect nominal exchange rate stability. The movements in the nominal exchange rate affect inflation, hence require a monetary policy intervention. Yet, intervention in the foreign exchange market may be inconsistent with the output gap and inflation target (Clarida et al., 2001; Corsetti & Pesenti, 2005).

The competing monetary policy objectives have reignited the debate on the effectiveness and optimality of monetary policy actions. Despite the raging debate, monetary policy operational frameworks and empirical studies have focused more on the effectiveness of monetary policy against price and output instability (for example, Kobayashi, 2004; Kholodilin et al., 2009; Koop et al., 2009; Misati et al., 2011; Misati & Nyamongo, 2012a). Yet, optimality of monetary policy stabilization actions is equally important in contemporary economies experiencing output and price shocks.

Therefore, this paper analyses optimal monetary policy stabilization of inflation, output, and asset prices in small open economies of Kenya and Ghana. The paper focuses on Ghana and Kenya because the economies have a developed financial sector and exhibit significant international portfolio flows, but they have different monetary policy frameworks. This enables the analysis of stabilization issue in open economies with comparable levels of development and openness, but under different monetary policy frameworks. In particular, the paper first establishes weights employed by monetary policy in response to inflation, output, and asset price volatility. The paper then evaluates optimality of the monetary policy response using a welfare criterion as in Divino (2009).

Hence, the paper develops a monetary policy reaction and social welfare functions incorporating volatility based on a dynamic stochastic general equilibrium (DSGE) model. The parameters of monetary policy reaction function are estimated using quarterly data from 2000 to 2018 for Kenya and Ghana. Other parameters of the model are calibrated to reflect characteristics of Kenyan and Ghanaian economies.

The social welfare function is solved numerically using the dynamic programming method of Bellman and Lee (1984), taking into account optimization decisions of households and firms; an approach that has been emphasized and used by Rotemberg and Woodford (1997), Benigno (2004), and Divino (2009) in their analysis of optimal monetary policy. The formulation and numerical solution of the social welfare problem as a dynamic programme enables estimation of the time path of monetary policy rate that maximizes social welfare. The path for optimal monetary policy rate incorporates static and dynamic responses of agents to monetary policy actions in an economy. The welfare, output, and price are then evaluated under discretion and consistency rules in the two economies.

The parameter estimates of the monetary policy reaction function indicate that inflation is accorded the strongest response of 1.48 and 1.41 compared to output of 0.25 and 0.12 for Kenya and Ghana, respectively. This is consistent with stated monetary policy objective of stabilizing prices to enhance growth in the two economies. Bond yields are stabilized by employing weights of 0.35 and 0.26, while equity prices have weights of 0.16 and 0.14 for Kenya and Ghana, respectively. The yields on bonds are stabilized with greater weight compared to equity prices. Stability of yields on bonds enables investors, especially financial institutions, to allocate credit and better manage liquidity through holding of liquid debt instruments in response to monetary policy action. This enhances the effectiveness of yields in transmitting monetary policy stimulus to the real sector via changes in asset prices. The responsiveness of monetary policy to equity and yields is greater for Kenya compared to Ghana. This is due to differences in the depth of the capital market in Kenya and Ghana, as well as strength of the pass-through of monetary policy stimulus to the real sector via changes in interest rate. Deeper capital markets are responsive to monetary policy and have a significant impact on the real sector. Consequently, monetary policy can target asset prices to stabilize price and output. These results are qualitatively consistent with estimates of Were (2014) and Bleaney et al. (2019) with respect to the response of monetary policy to inflation and output in Kenya and Ghana, respectively. The dynamic optimization results indicate that there are welfare gains when monetary authorities in Kenya and Ghana respond to inflation, output, and asset price instability consistently.

This paper, by estimating a reaction function and evaluating optimality of monetary policy action, makes the following contributions. First, the general equilibrium framework is able to capture the preferences of agents, as well as their responses to monetary policy. This provides a better estimate of sensitivity of monetary policy on the target variable. Secondly, the paper employs numerical methods to establish optimal response of monetary policy's objective function. Stability in prices and output stability are in the monetary policy's objective function. Stability in prices and output encourages investment and economic growth in developing economies like that of Ghana and Kenya. Thirdly, the study enriches the debate on stabilization problem faced by monetary authorities in small open developing economies, in which available tools are fewer than objectives to be achieved. More importantly, the paper brings to light

the stabilization outcomes in inflation targeting and in an economy transitioning to inflation targeting, an issue that has not received much attention. Fourth, the social welfare criterion for evaluating optimal monetary response is solved numerically using the dynamic programming method. This method enables evaluation of social welfare outcomes resulting from monetary policy intervention to stabilize output, asset prices and inflation under different regimes.

The rest of the paper is organized as follows. Section 2 briefly describes the conduct of monetary policy, and how it is used to stabilize output, inflation, and asset prices in Kenya and Ghana. Section 3 explains optimal monetary policy intervention in the economy. Section 4 and Section 5 discusses the methodology and data, and main results, respectively; while Section 6 summarizes issues from the main results. The section concludes that output and price objectives can be achieved optimally with a consistent and credible monetary policy.

2. Monetary policy in Kenya and Ghana

Monetary policy in Kenya and Ghana, like other developing countries, is mainly used to stabilize price, which is essential for encouraging investment and output growth. Besides price stability, monetary policy is also employed to mitigate output fluctuations. The evolution of monetary policy in the two countries has been shaped by economic reforms intended to hasten economic growth. Notably, liberalization and removal of price controls, between 1985 and 1995, necessitated prioritization of price stability over output growth in the implementation of monetary policy. This is because liberalization and price decontrols led to an increase in inflation, which undermined output growth. The emphasis on price stability objective achieved low and stable inflation, but at the expense of slow output growth rate between 1990 and 2001, whereby Kenya had a lower average growth of 2.1% compared to 4.1% for Ghana (see Figure 1).

Figure 1: Output, inflation and monetary policy





Source: Illustration using data from the Kenya National Bureau of Statistics and Ghana Statistical Service.

The monetary policy framework in Ghana and Kenya started with money targeting, whereby monetary authorities changed money supply in tandem with growth in nominal output. This ensures that there is just enough liquidity to finance transactions in the economy (McCallum, 1999). This explains the tendency for broad money to grow in tandem with inflation. The higher correlation between the growth rates of broad money and inflation in Kenya than in Ghana suggests that money targeting was more effective in controlling inflation in Kenya than in Ghana.

However, Ghana has transitioned fully to inflation targeting while Kenya is transitioning to inflation targeting. With respect to Ghana, monetary policy rate is adjusted following a Taylor rule in response to inflation stability. In addition, the monetary policy actions are geared towards influencing market interest rates and asset prices (Woodford, 2001; Kovanen, 2011; Bleaney et al., 2019). Monetary policy operations in Kenya are similar to those of Ghana insofar as they are market orientated. The difference is that, whereas monetary policy rate in Kenya is adjusted gradually towards the target rate in order to achieve inflation and output targets, the policy rate adjustment in Ghana is greater than the deviation of inflation from the target (Bleaney et al., 2019). The gradual adjustment of the policy rate to the desired target informs the financial market of the expected direction of monetary policy stance. This reduces uncertainty and builds inertia in the change in market interest rates, which enables economic agents to insure themselves against liquidity changes (Woodford, 2003, 2013).

The monetary policy reaction function in Ghana reduces uncertainty, because the interest rate rule informs expectations of the public about the reaction of monetary policy to inflation. However, the aggressive reaction to inflation leads to quick adjustment of market interest rates, which may precipitate liquidity distress to agents who cannot adjust their liquidity immediately. In addition, frequent changes in interest rates in response to inflation movements impede liquidity prediction, which reduces investment (Stulz, 1986; Wurgler, 2000). Yet, monetary policy in developing economies ought to encourage investment so as to realize economic growth and welfare improvement.

Another difference is that monetary policy in Ghana is precluded from responding to economic slowdown. This is because price stability objective supersedes output stability in the monetary policy reaction function. With respect to Kenya, movements in broad money is countercyclical to output growth rate, implying that monetary policy can be used to stabilize output (Vegh & Vuletin, 2012). Nevertheless, average output growth in Kenya is lower than in Ghana, but inflation is higher in Ghana compared to Kenya from 2000 to 2016 (Figure 1).

The level of openness of the economy and ease of portfolio flows through the capital market influence exchange rate movement. Significant movements in exchange rate disrupt allocation of liquidity and accentuate instability in the financial sector, which undermines investment and growth. This has informed Bank of Ghana and Central Bank of Kenya's intervention in the foreign exchange market to mitigate volatility in exchange rate. The changes in monetary policy rate in Ghana affect nominal exchange rate with a lag. In some periods, for example between 2013 and 2014, whereas monetary policy rate increased gradually, exchange rate fluctuated in a wider range. In addition, volatility in exchange rate increased in 2015 and 2016, suggesting that volatility in exchange rate. The increase despite an increase in monetary policy rate to stabilize exchange rate. The increase in exchange rate volatility may have been influenced more by commodity price shocks, especially cocoa. The ineffectiveness of monetary policy in stabilizing exchange rate is also exacerbated by a shallow financial market that inhibited transmission of monetary policy stimulus to real sector via changes in asset prices (Kovanen, 2011).



Figure 2: Effectiveness of monetary policy in mitigating exchange rate volatility



Source: Illustration using data from the Kenya National Bureau of Statistics and Ghana Statistical Service.

The monetary policy rate in Kenya was not effective in mitigating volatility in exchange rate between 2007 and 2010. The global financial crisis, as well political instability, may have undermined stabilization effort of monetary authorities. However, monetary policy was effective in stabilizing exchange rate between 2011 and 2018, as indicated by the deviation of exchange rate from its trend and being in concert with changes in monetary policy rate. The effectiveness of monetary policy in reducing volatility in exchange rate can be attributed to deeper financial market, which increases the responsiveness of exchange rate to liquidity changes induced by the Central Bank of Kenya.

A deep financial market is efficient in transmitting monetary policy stimulus to the real sector. Hence, the responsiveness of yields and equity prices to monetary policy rate influence effectiveness of monetary policy. The yields on Ghanaian bonds change in tandem with monetary policy rate. However, yields on bonds traded on the Nairobi Securities Exchange (NSE) in Kenya have a tendency to deviate from monetary policy rate and their fluctuations weakly mimic changes in monetary policy rate. This suggests that monetary policy in Ghana has a stronger influence on yields than in Kenya (Figure 3).





- Source: Illustration using data from the Kenya National Bureau of Statistics and Ghana Statistical Service.
- Notes: Yields are for government and corporate bonds traded on the Nairobi Securities Exchange (NSE) and Ghanaian Stock Exchange (GSE). The tenor for Kenya and Ghanaian bonds are 1–25 years and 1–7 years, respectively.

OPTIMAL MONETARY POLICY WITH INFLATION, OUTPUT AND ASSET PRICE VOLATILITY

Movements in equity prices in an efficient market can be induced by monetary policy. More importantly, monetary policy decisions influence equity prices in so far as they change the expected returns on investment, thereby influencing investors' decisions on the stock market (Bernanke & Kuttner, 2005, Funke et al., 2011). This has been exploited by monetary authorities to stabilize equity prices. In Figure 4, the cyclical component of Nairobi Stock Exchange All Share Index (NASI) prices for Kenya, and Stock Exchange-Composite Index for Ghana is counter-cyclical to monetary policy rate. However, monetary policy in Ghana has a stronger effect on equity prices thereby safeguarding the integrity of the capital market in allocating long term investment.





Source: Illustration using data from Kenya National Bureau of Statistics, Ghana Statistical Service, Ghana Stock Exchange (GSE), and Nairobi Securities Exchange (NSE). Note: The cycle for equity price indices is the Nairobi All Share index (NASI) for Kenya, and

Ghana Stock Exchange Composite index (GSE-CI) for Ghana.

Despite the fact that monetary policy in Kenya and Ghana is capable of enhancing price stability, there are differences in the responsiveness of equity, exchange rate, and yields on bonds, as well as the output growth outcomes in the economies. This calls for the analysis of not only the response of monetary policy target variables, but also optimality of monetary policy actions.

3. Optimal monetary policy stabilization of output and prices: Theory and evidence

The debate on stabilizing output and prices using monetary policy follows two broad strands. The first strand asserts that monetary authority should not intervene in the economy. The second strand advocates for monetary policy intervention to stabilize prices and output. The proponents of non-interventionist monetary policy argue that a market economy with flexible prices is self-regulating. This enables economic agents to face prices that provide incentive to produce output that maximizes individual and social welfare. Hence, monetary policy intervention in the economy is distortionary (Rotemberg & Woodford, 1997; Svensson, 1997).

However, shocks, market power of agents, as well as information asymmetry among agents during transactions distort prices. Price distortions either misallocate or lead to under-utilization of an economy's resources. This results in actual output and inflation deviating from potential output and inflation target, respectively. Even in an efficient market, equilibrium prices may not be socially desirable (Svensson, 1997; Benigno, 2004). Consequently, monetary policy intervention is required to remove price distortions that impede the achievement of socially optimum output and prices.

Even though Svensson (1997) and Benigno (2004) imply that monetary policy intervention is required to correct distortions in prices, they neither specify a set of prices to be targeted nor take into account instability that may result from policy intervention. Yet, monetary policy actions influence commodity and asset prices, as well as incentive to take risks, and hence financial stability. In addition, they do not specify a framework of monetary policy intervention in contemporary economies. Monetary authorities either change monetary aggregates in tandem with output growth or change policy rates following some rule, with the objective of enhancing price stability (McCallum, 1999; Woodford, 2013; Were, 2014). Woodford (2013) and Were (2014) show that rule-based monetary policy is effective in achieving inflation and output objectives.

However, rule-based monetary policy frameworks may not take into account agents' expectations and optimization decisions (Kydland & Prescott, 1977). As a result, monetary policy stabilization actions that follow some rule yield suboptimal welfare levels compared to stability achieved by a competitive market (King, 1997; Kydland & Prescott, 1977; Svensson, 1997). Nevertheless, a discretionary intervention achieves higher welfare than price or money targeting rule. This is because discretion affords monetary authority the flexibility to respond to unanticipated price and output changes, as well as dynamic optimization behaviour of economic agents. This is relevant for financial markets in which investors make decisions frequently to optimize their portfolio holding. In addition, small open developing economies are susceptible to unpredictable capital flows and terms of trade changes, which affect asset and commodity prices, respectively. Hence, there is need for monetary authority to apply discretionary monetary policy to address price movement.

However, discretion results in, first, higher volatility in either prices or output. This is because discretion does not provide agents with rules and information that they can use to make current decisions. Consequently, agents adjust their decisions as they get information, which affects stability of prices and output. Indeed, inconsistency in monetary policy action is a major cause of volatility in forward-looking markets (Svensson, 1997; McCallum, 1999; Woodford, 2003). Secondly, discretion allows monetary authority to use its superior information compared to the public to pursue its main objective. King (1997) argues that, monetary policy under discretion does not consider inflation expectations of the public when responding to output shocks. As a result, stabilizing output increases instability in inflation.

Inasmuch as Svensson (1997), Woodford (1994, 2001), Benigno (2004), and McCallum (1999) advocate for monetary policy intervention in the economy, they specify output and inflation as target variables. However, stabilizing asset prices and enhancing financial stability to achieve growth has gained credence in developing and developed economies. Käfer (2014) and Caporale et al. (2018) argue that monetary policy in developing economies strives to stabilize inflation, output, and asset prices, while the focus on nominal exchange rate depends on openness and vulnerability of domestic prices to foreign shocks.

Empirical analysis of optimal monetary policy intervention in the economy evaluates social welfare outcomes of monetary policy action. The analyses focus on the approaches that achieve price stability. This approach is based on the fact that when prices are stabilized at a level that enables a socially acceptable allocation and production, they lead to maximization of social welfare. For example, Rogoff (1985), Rotemberg and Woodford (1997), Svensson (1997), and King (1997) analyse the effectiveness of commitment and discretion in maintaining inflation stability. The studies establish that commitment to an interest rate rule, when responding to high inflation, leads to inflation stability, although output growth rate deviates from the target. The interest rate rule also leads to higher instability in output despite inflation and interest rate being stable. Woodford (2003) focuses on a consistent interest rate adjustment by monetary authority in response to inflation changes. In this analysis, smooth adjustment of interest rate in the expected direction informs the expectation of the public, who optimize based on the information available. The study establishes that interest rate adjustment with inertia has a higher welfare compared to surprise adjustments. The inertial adjustment of interest rate provides information required for formation of expectation about future asset price, which reduces uncertainty in forward-looking markets. Caporale et al. (2018) find that monetary authorities in developing and emerging markets accord more weight to inflation compared to

output and exchange rate. However, output and exchange rate are accorded more weight if inflation is either within the target or there are no threats to price stability.

Clarida et al. (2001), Benigno (2004), Corsetti and Pesenti (2005), and Divino (2009) extend the analysis to include exchange rate. In this way, they consider distortions in the domestic prices emanating from foreign economies. Their analytical solution of the social welfare function establishes that a consistent interest rate adjustment to stabilize inflation leads to output instability, because changes in interest rate affect exchange rate, which then causes a deviation of output from the socially desirable level. In this case, stabilizing inflation distorts exchange rate, which reallocates resources in a manner that is socially undesirable. Recent studies by Fujiwara and Wang (2017) focusing on interaction between two monetary policy authorities find that there are welfare gains from optimal response to monetary policy inflation output and exchange rate.

Whereas monetary policy affects risk taking and asset prices in the financial market, empirical studies have focused more on stability of inflation, output, and nominal exchange rate. Woodford (2012) and Käfer (2014) find that imbalances in asset prices can be corrected by augmenting a monetary policy response function with a measure of financial stability. Indeed, Christiano et al. (2010) find that increasing interest rate over and above the level required to stabilize inflation corrects excess liquidity used to exacerbate distortions in asset prices. Proximate studies on Ghana, such as Kovanen (2011) and Bleaney et al. (2019), find that monetary policy has insignificant impact on interest rate; while Misati et al. (2011) and Were (2014) show that the impact of monetary policy on interest rate and equity prices is weak in Kenya.

There are few limitations to these analyses. Firstly, they do not take into account financial asset prices, like bond and equity prices, when monetary policy is responding to price distortions. The equity and bond prices are one of the channels through which monetary policy stimuli is propagated in the real sector via the financial sector. Therefore, a monetary policy response function without equity and bond prices, which result from optimization decision of agents on the financial market, is deficient of important information (Käfer, 2014). More importantly, asset price movements affect aggregate demand by changing the net-worth and wealth of firms and households. Fluctuations in aggregate demand, as a result of asset price, induce instability in output.

Furthermore, asset price instability stifles growth by undermining the integrity of the financial sector to mobilize and distribute capital in the economy by distorting return on capital and balance sheets. Therefore, monetary authority eager to stabilize output and enhance growth, especially in developing economies, has to include asset prices in its objective function. Secondly, equity and bond prices are a source of information that can be used by monetary authority to stabilize inflation (Bernanke & Gertler, 2000; Mishkin, 2001). Furthermore, monetary policy rate can inform pricing of risk on the yields of bonds, thereby influencing the term structure and the yield curve. Finally, in the general analytical solutions of Divino (2009), Clarida et al. (2001), Corsetti and Pesenti (2005), and Benigno (2004), social optimization problem with monetary policy intervention does not yield numerical results that can be compared.

Therefore, this paper tries to fill these gaps by analysing optimal monetary policy response to output and asset price volatility. The paper develops a dynamic stochastic general equilibrium model of an open economy with asset prices, and then calibrated to capture unique features of Kenyan and Ghanaian economies. Other parameters of the model are then estimated. A welfare analysis approach is used, because optimal intervention is more effectively evaluated by comparing welfare outcomes of alternative interventions. In addition, the welfare approach takes into account the optimization decision of the public, which is relevant for monetary policy. The social problem is also solved using dynamic programming numerical method, which allows for a quantitative comparison.

4. The model and data

The monetary policy stabilization actions influence interests and yields, which then affect portfolio decisions, and hence consumption and production. This influences prices and output in the direction desired by monetary authorities (Misati et al., 2011; Misati & Nyamongo, 2012b). Underpinning price and output changes towards the monetary policy target is the change in the optimal decisions of agents. Households optimize their portfolio holding to maximize utility taking price as given, while firms adjust prices Calvo (1983) style to maximize profits. The Calvo (1983) style price adjustment captures price setting behaviour of firms in Kenya and Ghana. The economies have some firms with market power, while other firms take price as given and decide on the quantity to produce.

Holding assets is risky and an average household is risk averse. Hence, a constant relative risk aversion utility function suffices in describing utility derived from consuming goods and service as well as holding financial assets. More importantly, changes in money balances held by the public induced by movements in inflation and asset prices affect liquidity services derived from money. Besides liquidity services, inflation and asset prices affect the amount of wealth, and hence consumption of goods and services (Funke et al., 2011). Therefore, the utility function of a representative household is best described by:

$$V_{t} = E_{t} \sum \beta^{t} \left[\frac{C_{t}^{1-\sigma}}{1-\sigma} - \frac{N_{t}^{1+\gamma}}{1+\gamma} + \frac{(M_{t}/P_{t})^{1-\eta}}{1-\eta} \right] V_{t} = E_{t} \sum \beta^{t} \left[\frac{C_{t}^{1-\sigma}}{1-\sigma} - \frac{N_{t}^{1+\gamma}}{1+\gamma} + \frac{(M_{t}/P_{t})^{1-\eta}}{1-\eta} \right]$$

Where: C_t is consumption, M_t/P_t is real money, N_t is labour, and β the discount rate. Parameters σ are the coefficient of relative risk aversion with respect to consumption, γ is the inverse of the Frisch labour supply elasticity, η is the coefficient of relative risk aversion with respect to money balances, and E_t is the expectation operator.

$$P_{t}C_{t} + E_{t}\left[\Psi_{t,t+1}B_{t+1}\right] + P_{t}\int_{0}^{1}Q_{t}(i)Z_{t}\partial i + I_{t} + \frac{M_{t+1}}{(1+i_{t})P_{t+1}} = W_{t}N_{t} + r_{t}K_{t} + B_{t} + P_{t}\int_{0}^{1}[Q_{t}(i) + D_{t}(i)]Z_{t+1}\partial i + \frac{M_{t}}{P_{t}}$$

$$(2)$$

(1)

Households maximize utility given by Equation 1 subject to the budget constraint (2). Where, $\Psi_{t,t+1} = 1/(1+i_t)$ is the nominal gross return on bonds, W_t wage rate, B_t bonds, Z_t equity shares, K_t capital, Q_t equity price, and r_t real interest rate. I_t is investment given by $I_t = (1 - \delta)K_t - K_{t-1}$. Adjusting prices with nominal exchange rate, ε_t , and obtaining first order conditions gives:

$$C_{t} = \left[\beta \left\{ \frac{P_{t}}{P_{t+1}} \frac{\varepsilon_{t}}{\varepsilon_{t+1}} (\mathbf{3}) + r_{t+1} - \delta \right\} \right]^{-\frac{1}{\sigma}} C_{t+1}$$
(3)

$$N_t = \left[C_t^{-\sigma}W_t\right]^{\frac{1}{\gamma}} \tag{4}$$

 $\Psi_t = 1 + r_{t+1} - \delta \tag{5}$

$$M_t^{\eta} = -\frac{c_t^{-\sigma}}{\varepsilon_t P_t} + \beta \frac{c_{t+1}^{-\sigma}}{(1+i_t)\varepsilon_{t+1} P_{t+1}}$$
(6)

$$Q_{t} = \left[\frac{P_{t+1}}{P_{t}}\tilde{\varepsilon}_{t+1}(Q_{t+1} + B_{t+1})\right] / 1 + r_{t+1} - \delta$$

$$Q_{t} = \left[\frac{P_{t+1}}{P_{t}}\tilde{\varepsilon}_{t+1}(Q_{t+1} + B_{t+1})\right]/1 + r_{t+1} - \delta$$
(7)

The inclusion of the nominal exchange rate provides a conduit for foreign shocks to affect the domestic economy. Equations 3–7 define optimal consumption, labour supply, yields on bonds, money demand, and equity prices, respectively.

Firms produce output Y_t using Cobb Douglas production technology: $y_t = e^{A_t} K_t^{\alpha} N_t^{1-\alpha}$; where, capital K_t , labour N_t , $A_t = \rho A_{t-1} + \varrho_t \epsilon$, and $\varrho_t = \varpi \varrho_{t-1} + v_t$; and $\varrho_t = \varpi \varrho_{t-1} + v_t$; and $\varrho_t = \varpi \varrho_{t-1} + v_t$; so the variance of technology shock ϵ . Competitive firms maximize profit by choosing capital K_t and labour, while firms with market power set current price, P_t , to maximize discounted profit. The price setting decisions of firms is described by Equation B15 in Appendix B.

The optimization decision of the household and firm establish aggregate supply given by:

 $\begin{aligned} y_t &= E_t y_{t+1} - \frac{\beta}{\sigma} [\{i_{b,t}\} - E_t \tilde{\varepsilon}_{t+1} - E_t \pi_{t+1}] \end{aligned} (8) \\ \text{Where: } \pi_t &= \frac{(1-\Theta)(1-\Theta\beta)}{\Theta} mc_t + \beta E_t (\pi_{t-1}) \text{ is the Philips curve obtained from price setting decision of firms, } i_{b,t} \text{ the interest rate, } y_t \text{ is the output, } \tilde{\varepsilon}_{t+1} \text{ the change in exchange rate, } \pi_t \text{ inflation, and } mc_t \text{ the marginal cost. The change in exchange rate is an autoregressive process, with a stochastic variance. The derivations for aggregate the stochastic variance. The derivation of the stochastic variance th$

supply and the Phillips curve functions are given in the Appendix A.

From the aggregate supply function and the Philips curve, the exchange rate influences output via two channels. The first channel is through changes in aggregate supply as a result of change in domestic demand. In addition, exchange rate movements alter balance sheets of firms and households, which, not only affect credit worthiness, but also wealth, and hence aggregate demand (Mishkin, 2001; Divino, 2009). The second channel is through marginal cost of producing goods and services. In small open economies, with flexible exchange rate regime, exchange rate movements have a significant effect on domestic marginal costs as it influences real wage and cost of intermediate imported goods. Interest rate, $i_{b,t}$, affect yields on bonds as well as return on investment, and hence equity prices. In this case, changes in interest rates influence aggregate demand by affecting asset prices. Making interest rate $i_{b,t}$ the subject, and using aggregate supply function (8), yields a monetary policy response function given by:

$$i_{b,t} = \frac{\sigma}{\beta} \left(E_t y_{t+1} - y_t \right) - \frac{\beta}{\sigma} E_t \left[\tilde{\varepsilon}_{t+1} - \pi_{t+1} \right]$$
(9)
Therefore, the central bank can achieve output and inflation objectives by changing

the policy rate, taking into account output gap, expected exchange rate depreciation, and inflation. The changes in interest rate, according to Equation 9, minimize mc_t , inflation, exchange rate, and output gap deviations from zero (Woodford, 2003; Gali & Monacelli, 2005).

An optimal monetary policy maximizes social welfare, given the optimal decision of the public. Hence, changes in prices occasioned by a monetary policy response function described by Equation 9 may not be optimal. This is because changes in interest rate informed by Equation 9 do not take into account social preference of the public with respect to inflation, asset holding, and consumption (Lucas & Stokey, 1983). Therefore, the price level that allows individuals to optimize their consumption, leisure, and financial asset holding, leads to social welfare optimization, as depicted by the social preferences. A proximate social welfare function is the second order Taylor approximation of the household utility function (Rotemberg & Woodford, 1997). Divino (2009) and Woodford (2003) use a similar approach. In this regard, a second order Taylor approximation around the steady state of Equation 1 is used to approximate the social welfare function. This is the quadratic loss function given by Equation 10. The social loss function captures the effect of monetary policy on social welfare, as a result of changes in money balance and asset prices.

$$L_t = \sum_{t=0}^{\infty} \beta^t \left(\gamma_\pi \pi_t^2 + \gamma_y y_t^2 + \gamma_Q \tilde{Q}_t^2 + \gamma_B \tilde{B}_t^2 \right)$$
(10)

$$y_{t} = E_{t}y_{t+1} - \frac{\beta}{\sigma} [\{i_{b,t}\} - E_{t}\tilde{\varepsilon}_{t+1} - E_{t}\pi_{t+1}]$$
(11)

$$\pi_t = \frac{(1-\Theta)(1-\Theta\beta)}{\Theta} mc_t + \beta E_t(\pi_{t-1})$$
(12)

$$Q_{t} = \left[\frac{P_{t+1}}{P_{t}}\tilde{\varepsilon}_{t+1}(Q_{t+1} + B_{t+1})\right] / 1 + r_{t+1} - \delta$$
(13)

$$\Psi_{t+1} = [1 + r_{t+1} - \delta] \tag{14}$$

Where: L_t is the social loss to be minimized; γ_{π} , γ_{ν} , γ_{o} , γ_{B} are contributions of inflation, income, equity price, and debt instability to social loss, respectively; \tilde{B}_t is the stock of debt; y_t output; y_t inflation; E_t is the expectations operator, \tilde{Q}_t equity price; β discount factor; $\tilde{\varepsilon}_t$ change in nominal exchange rate; and t is time. The details of derivation are provided in Appendix B. Indeed, Equation 10 depicts preferences of the public with respect to inflation, output, equity, and bond prices, which also affect consumption, leisure, and money balances. An optimal monetary policy minimizes deviation of output, inflation, interest rate, bond, and equity prices from their equilibrium values. This entails minimizing Equation 10, subject to the first order condition of the utility maximization problem of the public given by equations 11–14. Equation 11 is the Lucas aggregate supply function; Equation 12 describes change in inflation over time; Equation 13 and Equation 14 are the equity price and debt stock equations, respectively, obtained from the household optimization decision. This specification takes into account the fact that monetary policy cannot fully influence rational expectations of the public. This affects their optimization decisions in relation to monetary policy actions.

Following Lucas and Stokey (1983), monetary authority in this case chooses a policy rate that enables an allocation of credit that maximizes social welfare given the resource constraint and the decisions of agents in the economy. In particular, a monetary authority chooses a target interest rate level and decides on the time path the policy interest rate should follow to stabilize inflation. The time path of the monetary policy rate then influences the time path of the market interest rate and the nominal exchange rate. In a small open economy with portfolio flows, movements in domestic interest rates relative to foreign interest rate influence returns on assets. The resultant differential in asset returns provide an opportunity for arbitrage, which induces changes in exchange rate.

In addition, monetary policy geared towards influencing market interest rates and yields on bonds, is underpinned by changes in liquidity, especially domestic currency in circulation. The changes in composition of domestic currency relative to foreign currency influence nominal exchange rate. Therefore, an optimal monetary policy intervention requires the entire path of monetary policy interest rate to minimize the social loss function. Thus, the solution a monetary authority provides to the social optimization problem depends on the monetary policy rule used to respond to instability in variables in the social welfare function.

A commitment to policy rule entails solving optimization problem, and the obtained solution becomes a rule for responding to deviation of variables of interest from the desired path over time. In particular, the monetary authority commits to an interest rate rule to minimize L_t . Thus, the problem can be recast as a dynamic optimization

problem, whereby L_t is a state variable and monetary policy rate is the decision variable. Therefore, under commitment, the central bank commits credibly to a particular rule when intervening in the market to stabilize price such that L_t is the lowest. However, under discretion, Equation 10 is solved subject to the private sector's optimization without commitment to a policy rule. This specification allows the application of dynamic programming method of Bellman and Lee (1984) to solve for the optimal interest rate path.

The horizon of the optimal path after a shock depends on either expectations or the persistence of the shock in the data. Some agents may form adaptive expectations about a monetary policy response to output and price instabilities based on the effectiveness of the past actions. Other agents evaluate credibility of the monetary authority's commitment to the achievement of its main target for over slightly a year. In addition, monetary policy in most economies has an impact that ranges between the first and the fifth quarter. Hence, optimizing a social welfare function for over five quarters by choosing an optimal path of a monetary policy rate is appropriate. However, some shocks are quite persistent and, therefore, the horizon of optimization and shocks are selected based on the persistence of shocks in the underlying data generating process as in Fujiwara and Wang (2017). The horizon based on the data generating process is more informative.

Even though dynamic optimization has also been used by Woodford (2003) and Wamalwa (2018), Söderlind (1999) argues that, optimal response can be estimated using regression methods. However, regression estimates of the quadratic loss function parameters are average weights that do not reflect optimization behaviour of the central bank and the public as well as the reaction of agents to policy decision (Rotemberg & Woodford, 1997). In this regard, the estimates are based on quarterly data for Kenya and Ghana between the first quarter of 2000 and the second quarter of 2018.

The generic model consisting of equations 1–10 by construction is the same for Kenya and Ghana except for differences in calibration of parameters to reflect country specific characteristics. Other model parameters are estimated using country data. Therefore, the optimal monetary response is comparable in the two countries. Table 1 presents parameters calibrated for the model to reflect country characteristics.

Parameters	Definition		Ghana	Kenya
σ	Elasticity of consumption risk aversion	Calibrated	0.9	0.8
γ	Inverse of the Frisch elasticity of labour supply	Calibrated	0.9	1.0
η	Elasticity of money demand	Calibrated	5	4.5
β	Discount factor	Calibrated	0.923	0.95
θ_{yt}	Sensitivity of central bank to output gap	Calibrated	0.035	0.25
$\theta_{\pi t}$	Sensitivity of central bank to inflation	Calibrated	1.5	1.3
θετ	Sensitivity of central bank to exchange rate	Calibrated	0.25	0.25
Θ	Fraction of firms with market power	Calibrated	0.698	0.698
φ	Degree of openness	Calibrated	1.4	1.4
α	Share of capital in output	Calibrated	0.33	0.34

Table 1: Parameters

Notes: This table indicates parameters for the models.

Non-food non-fuel is included because monetary authorities target non-food nonfuel inflation in bid to regulate aggregate demand. Even though bond prices can be used in the model instead of yields, this paper utilizes yields because their movements are closely related to money market interest rates, which is an intermediate target of monetary policy. In particular, monetary policy in Kenya and Ghana targets marketbased intermediate variables such as interbank interest rate, which in turn influence retail interest rates and yields on bonds. Hence, including yields in the model takes into account the effect of monetary policy to the real economy via changes in interest rate. Equity prices in Kenya and Ghana are measured by the Nairobi All Share index (NASI) and Ghana Stock Exchange Composite Index (GSE-CI), respectively.

5. Results

Table 2 presents Bayesian estimates of the dynamic stochastic general equilibrium model based on data for Kenya and Ghana. The parameters indicate the response of the variable to the change in monetary policy rate. Thus, the coefficient of the estimated quadratic loss function is the penalty the monetary policy imposes on the deviation of the variable in the social welfare function from the target or its long-run level. Volatility is captured by estimating a third order Taylor expansion of the model, as well as the variance processes of the quadratic loss function. The prior and posterior distribution of parameters estimated is provided in Figure 5. The distribution of the estimated parameters converges to the mode.



Figure 5: Prior and posterior distribution

Source: Illu\stration using data from Kenya National Bureau of Statistics and Ghana Statistical Service.

Notes: The grey, black, and light green lines indicate distributions of the prior, posterior, and the mode, respectively. phi_exch is exchange rate, phi_ib is yields on bonds, phi_pi is the inflation, phi_y is output, and phi_iq is the equity. Brooks and Gelman (1998) convergence diagnostics test in Figure B1 and Figure B12 (in Appendix B) indicate that the sampling was random and there is convergence between the data and the simulated theoretical distribution as the sample size increases. The moment from posterior distribution also tend to 1 as the number of iterations increase. This indicates that the information from the data is combined with the priors to obtain valid estimates. Table 2 presents the prior and posterior **means for parameter estimates**.

	Prior Mean	Posterior Mean	90% HPD Interval		Prior Distribution	pstdev
Kenya						
Inflation	1.500	1.4816	1.4658	1.4980	gamma	0.01
Bond yields	0.350	0.3515	0.3384	0.3715	gamma	0.01
Equity price	0.150	0.1040	0.0950	0.1112	gamma	0.11
Exchange rate	0.250	0.2207	0.2065	0.2354	normal	0.01
Output	0.350	0.3506	0.3331	0.3681	gamma	0.01
Ghana	Prior Mean	Posterior Mean	90% HPD In	terval	Prior Distribution	pstdev
Inflation	1.500	1.4983	1.4821	1.5151	gamma	0.01
Bond yields	0.350	0.3551	0.3353	0.3727	gamma	0.01
Equity price	0.150	0.0797	0.0615	0.0934	gamma	0.01
Exchange rate	0.250	0.2492	0.2337	0.2684	normal	0.01
Output	0.350	0.3484	0.3302	0.3659	gamma	0.02

Table 2: Quadratic loss function

Source: Computation based on data from Kenya National Bureau of Statistics, Ghana Statistical Service, NSE, and GSE.

The results in Table 2 indicate that monetary authorities in both countries accord greatest weight to inflation as indicated by the inflation parameter. This is consistent with price stabilization objective of monetary policy in small open economies. Despite the fact that the responsiveness of monetary policy in Kenya and Ghana to inflation is greatest, Ghana has a more robust response than Kenya. This is expected of inflation targeting regimes in which monetary policy is more responsive to inflation than in money targeting regimes. The differences could also be attributed to Ghana having a higher inflation target of 8±2% compared to a target of 5±2.5% for Kenya.

Short-term interest rates and yields on bonds allocate liquidity in the financial market. Hence, an effective monetary policy affects the direction and distribution of funds by influencing yields and interest rates (Ennis & Keister, 2008). However, nascent financial markets have rigidities that impede the impact of monetary policy rate on

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yields. Therefore, effective monetary policy minimizes the deviation of yields from the monetary policy rate, which is, not only the target, but also an anchor (Kovanen, 2011). A unit deviation of bond yields from the target leads to change in monetary policy rate of 0.352% and 0.256% for Kenya and Ghana, respectively. Monetary policy actions influence interest rate on the money market, which in turn affect yields, and hence pricing of bonds in the secondary market. The change in bond prices affects aggregate demand through changes in wealth. As a result, yields on bonds constitute an intermediate target for monetary policy. More importantly, developing economies like Ghana and Kenya finance their fiscal deficits by selling bonds on the financial market, which influences yields on the secondary market. This influences stability of bonds prices, as well as the effectiveness of the asset price channel of monetary policy transmission mechanism (Di Bartolomeo & Giuli, 2011). Furthermore, movements in bond yields induce portfolio adjustment, which drive asset prices. Hence, monetary authorities take into account stability of yields on bonds to enhance stability of asset prices and effectiveness of monetary policy. The pass-through effect of monetary policy to the yields is weaker in Kenya compared to Ghana. The differences in the response of monetary policy to yields can be attributed to the relative impact of monetary policy stimulus to interbank and money market interest rate and then the yields on bonds.

The response of monetary policy to equity prices is larger in Kenya compared to Ghana. This can be due to differences in the size of market capitalization in relation to the economy. An efficient equity market establishes asset prices that are consistent with discounted present value of dividends. This enables investors to allocate investible funds efficiently, thereby hastening capital accumulation and economic growth. In addition, fluctuations in equity prices influence value of collateral as well as wealth, which inform borrowing and consumption decision. Consequently, equity price misalignment may encourage suboptimal debt accumulation, thereby predisposing borrowers to financial distress (Botzen & Marey, 2010; Borio & Lowe, 2002). This also undermines the quality of loans and profitability of financial institutions. Indeed, financial distress as a result of asset price misalignment has been cited as one of the causes of financial sector and output instability. Therefore, monetary authority in developing countries takes into account equity prices, not only to enhance financial sector stability, but also output growth.

Nominal exchange rate provides a proximate conduit through which foreign prices in Kenya and Ghana influence domestic prices (Divino, 2009). As a result, monetary policy considers stability of nominal exchange rate when responding to inflation, asset prices, and output instability. Monetary authorities in Kenya and Ghana attach weights to nominal exchange rate stability of 0.217% and 0.250%, respectively. This may be due to Ghana's economy being more open, which predisposes domestic prices to foreign price shocks. As a result, for a unit nominal exchange rate deviation from the trend, monetary policy rate is changed by 0.22% and 0.25% for Kenya and Ghana, respectively. This suggests a gradual adjustment of the nominal exchange rate towards the trend. An intervention in the foreign exchange market that builds inertia in exchange rate adjustment towards the target reduces volatility in the exchange rate. This is consistent with managed floating exchange rate regimes in which monetary authority minimize fluctuations in the exchange rate.

Monetary policy rate changes by 0.25% and 0.13% in Kenya and Ghana, respectively, in response to output growth deviating from the growth of the output trend. The response to output fluctuation is accorded the least weight in Ghana. However, Kenya's monetary response to output is larger compared to Ghana. Whereas Ghana and Kenya are developing economies that need to utilize monetary policy to enhance growth, price stability is prioritized. This is because output stability has the least weight for Ghana, while in Kenya output weight is only exceeded by inflation and yields on bonds. The differences in prioritizing output are reflected in the actual output growth, where Kenya has a higher average growth of 5.7% compared to Ghana with a growth rate of 4.3%. The implication of this result is that the framework employed by the Bank of Ghana prioritizes inflation. This enables market mechanism to allocate resources efficiently, which accelerates capital accumulation and growth. However, price stability is achieved by retaining interest rate at high levels. This stifles borrowing for investment, which is a growth catalyst. Indeed, yields in Ghana averaged 22.02%, while in Kenya it averaged 9.70% between 2013 and 2018. The monetary policy outcomes entailing higher growth and higher inflation in money targeting regimes compared to low growth and low inflation in inflation targeting is consistent with the predictions of McCallum (1999) and Woodford (2013, 2001).

The impulse response functions for monetary policy shocks are consistent with the response of monetary policy to inflation, interest rate, equity, and bond prices. A 1% shock increase in the monetary policy rate reduces inflation by about 1.3% in Kenya, while inflation in Ghana reduces by 3.8%. The differences in the responses of inflation to the same size of shock could be due to inflation targeting framework than reinforced monetary policy stimulus. The nominal interest rate in Ghana is also more responsive than in Kenya. As a result, the bond prices reduce by 1.4 points in Ghana compared to about 0.5 points in Kenya. Equity prices increase by about 20 points and 6 points for Ghana and Kenya, respectively, due to the decline in bond prices. Output growth declines by 0.4 percentage points in Kenya, while in Ghana growth declines by about 1.8 percentage points as a result of a monetary policy shock, while exchange rate depreciates (Figure 6).



Figure 6: Impulse response function

Notes: eps is exchange rate, ib is bond prices, pi is inflation, y is the output, and q is equity price index,

The implication of these results is that an aggressive response to inflation decelerates output growth faster than a less aggressive response. Despite the fact that exchange rate is one of the target variables, inflation, output, equity, and bond price stabilization seems to undermine stability of exchange rate. This is due to monetary authority having few instruments than the target variables. However, the optimality of the monetary policy action is even more pertinent because exchange rate influences domestic prices and output. Optimal response is discussed in the subsection hereunder.

Optimality of monetary policy action

The preceding subsection analysed the response of monetary policy to output, exchange rate, inflation, equity price indices, and yields. However, the effectiveness of monetary policy in an economy with competing monetary policy objectives depends on optimal response to variables of interest, as well as taking into account the response of the public. In particular, monetary policy achieves inflation and output gap targets if inflation and output expectations of the public are properly anchored.

In this regard, monetary authority responds to current output, inflation, and asset price instability considering the current and future expectation of the public. Under discretion, the monetary authority optimizes the welfare function for each period. Figure 7 presents the optimal monetary policy rate and actual policy rate obtained from solving a dynamic program of the monetary authority outlined by equations 10–14. The optimal path for the monetary policy rate is solved using numerical methods by minimizing the quadratic loss function each period. The dynamic

responses to inflation in Figure 7 indicate that optimal monetary policy rate exhibit wide fluctuations, however inflation is lower in both countries. In addition, fluctuations in optimal inflation and optimal monetary policy rate are in concert for Ghana. This implies that responding to price shocks taking into account expectation of the public results in lower inflation. Despite the fact that optimal inflation is lower and fluctuates in tandem with optimal monetary policy rate, optimal inflation and monetary policy rate are more volatile than actual inflation and policy.





Optimal inflation

Source: Illustration using data from Kenya National Bureau of Statistics and Ghana Statistical Service.

Optimal policy rate

Actual policy rate

Actual inflation

The optimal monetary policy rate results to optimal output growth rate of 5.6% and 4.12% for Ghana and Kenya, which is lower than actual average growth rate of 5.7% and 4.3%, respectively. This implies that the path for the optimal rate is higher

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than the actual policy rate, exerting a contractionary effect on the output growth that undermines achieving output growth rate objective. Inasmuch as the path of monetary policy interest rate achieves a lower average inflation for the two countries, it is too high to be consistent with the higher income growth rate that is essential for welfare maximization. The high interest rate reduces returns to investment that discourage investment in the economy. This leads to low output growth rate. In addition, optimal policy rate is more volatile compared to actual policy rate in both countries. This induces volatility in inflation. Volatile prices induce uncertainty in returns to investment that discourage not only investment but also transactions, thereby undermining efficient functioning of markets (Malkiel, 2003; Lester et al., 2012).

One of the solutions to inconsistent optimal interest rates with respect to achieving monetary policy objective is setting monetary policy target and responding to shocks in monetary target variables consistently (Woodford, 2003). Therefore, the social loss function (Equation 10) is minimized, subject to equations 11–14, taking into account inflation targets of 8±2% and 5±2.5% for Ghana and Kenya, respectively, for over six quarters. The results are shown in Figure 8. The horizon of six quarters is based on the empirical evidence that the financial sector and the real sector responds to monetary policy within five to six quarters (Kovanen, 2011; Misati & Nyamongo, 2012a; Were, 2014).



Figure 8: Commitment



Source: Illustration using data from Kenya National Bureau of Statistics and Ghana Statistical Service.

The path for optimal monetary policy in Kenya and Ghana achieves lower and stable inflation compared to discretionary monetary policy in Figure 7. However, average optimal policy rate is higher than actual policy rate. The commitment to achieve inflation objectives as well as a consistent and credible monetary intervention, by keeping monetary policy rate high, anchors inflation expectations. The anchored inflation expectations, not only inform portfolio allocation decisions, but also influence aggregate demand (Lester et al., 2012). This increases effectiveness of monetary policy. The social loss value is 235.9 for Kenya and 279.4 for Ghana with commitment to an optimal rule, which is lower compared to welfare value of 11875.1 for Ghana and 5763.1 for Kenya under discretion. This implies that commitment to a monetary policy rule enables the society to attain a higher welfare compared to monetary policy rule under discretion. The results are consistent with King (1997), Kydland and Prescott (1977), Svensson (1997), and empirical analysis of Bailliu et al. (2015), with regards to reaction of monetary policy to financial market imbalances. The effectiveness of commitment to monetary policy rule stems from certainty of monetary policy action, which reduces instability in prices. Stable price encourage investment, hence the higher output growth of 4.2% for Kenya and 5.8% for Ghana.

However, equity price cycles under commitment have an average variance of 363.5 and 30.7 for Ghana and Kenya, respectively. This is greater than variance under discretion of 337.3 and 0.6 in GSE-CI and NSE index, respectively. This implies that, whereas equity prices are responsive to monetary policy, persistence of stability causes volatility in asset prices. Furthermore, the fluctuations in optimal GSE-CI tracks well the actual GSE-CI albeit with a lag. However, the NSE cyclical component is weakly influenced by the monetary policy (Figure 9).







Source: Illustration using data from Kenya National Bureau of Statistics, Ghana Statistical Service, NSE, and GSE.

Notes: GSE-CI[NEED TO MAKE LABEL CLEAR IN THE FIGURE] is the GSE-CI index cycle, GSE-CI_optimal is the optimal GSE-CI under discretionary monetary policy, and GSE-CI_ optimal_commitment is the GSE-CI cycle under commitment. NSEALL_actual is the NSE index cycle, NSEALL_optimal_dis is the optimal NSEALL under discretionary monetary policy, and NSEALL_optimal_commitment is the NSEALL cycle under commitment.

The results suggest that, price and output stability increases volatility in equity prices. This is consistent with the finding of Botzen and Marey (2010) and Mishkin (2001) that, stability in inflation and output growth builds misaligned equity prices

from the discounted dividends, which exacerbate volatility in the financial market. The weak impact of monetary policy on equity prices in Kenya is consistent with results on the response of monetary policy rate to equity prices in Table 2. The implication of the results is that, the propensity of equity prices influencing financial sector stability and domestic prices is lower in Kenya than in Ghana.

6. Conclusion

This paper focused on optimal monetary policy response to output and financial asset price instability. The results from dynamic stochastic general equilibrium (DSGE) model indicate that, monetary policy responds more strongly to inflation, followed by interest rate, yields, equity prices and finally output. In this regard, the response is consistent with the main objective of stabilizing prices. Stable prices enable market mechanism to allocate resources to efficiently produce goods and achieve growth, which is the ultimate monetary policy objective in developing economies. Whereas price stabilization is the stated objective of monetary authorities in Kenya and Ghana, there are differences in the output and inflation outcomes. Namely, Kenya has higher weight on inflation, but has lower average inflation and higher output growth compared to Ghana, which employs inflation targeting framework. In addition, inflation and interest rate are higher in Ghana than Kenya. This implies that prioritizing price stability induces contractionary effect on growth in Ghana.

The effectiveness of monetary policy depends on the reaction of the public to a monetary policy stimulus. Hence, an optimal monetary policy maximizes social welfare by aligning social preferences with regards to output, asset prices, and inflation with monetary authority's preferences. The results indicate that, social welfare is maximized when monetary policy responds to inflation instability more strongly compared to output growth rate deviating from the desired target. Monetary policy should also respond to distortions in the equity and bond prices with less weight compared to exchange rate. In addition, a consistent monetary intervention in the economy has lower social loss than discretionary intervention. This implies that, monetary policy actions that minimize uncertainty in the economy enhance liquidity management, optimal portfolio holding, and efficient investment, which are essential for growth. The emphasis on price stabilization objectives in inflation targeting regime in Ghana seem to be competitive with output objective.

However, under welfare criterion, stabilizing prices and output consistently is complementary, which increases welfare. Therefore, in developing economies, stabilizing prices and output, taking into account the response of the public, increases the effectiveness of monetary policy and welfare. The implication of these results is that price and output stability outcomes are influenced by the manner in which monetary policy is used and not the monetary policy frameworks. In addition, taking into account all prices and output increases effectiveness of monetary policy as well as welfare. Therefore, the response of monetary policy to exchange rate, equity prices, bond prices, and output does not prejudice inflation stability. Despite the fact that the DSGE model captures the effect of monetary policy in details and more accurate than competing frameworks such as general equilibrium and Vector Autoregression frameworks, the model did not include bond issuing behaviour of corporate and governments. The auction and pricing mechanism of bonds affect monetary policy and yields on the stock market. In addition, changes in yields affect equity prices. Therefore, a possible extension of this paper should include bond issuing equations for corporate and governments.

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Appendixes

Appendix A

This appendix derives households' and firms' optimization problem, as well as price setting behaviour of firms with substantial market power. First, the appendix explains household utility maximization decision, whereby a representative household maximize a CRRA utility function consisting of consumption C_t , money M_t , and labour N_t assets.

$$V_t = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} + \frac{(M_t/P_t)^{1-\eta}}{1-\eta} \right]$$
(A1)

Subject to a budget constraint:

$$\begin{split} P_t C_t + E_t \Big[\Psi_{t,t+1} B_{t+1} \Big] + P_t \int_0^1 Q_t(i) Z_t \partial i + I_t + \frac{M_{t+1}}{(1+i_t)P_{t+1}} &= W_t N_t + r_t K_t + B_t + \\ P_t \int_0^1 [Q_t(i) + D_t(i)] Z_{t-1} \partial i + \frac{M_t}{P_t} \end{split}$$

(A2)

(A3)

Where: $\Psi_t = 1/1 + i_{bt}$ is the nominal gross return on bonds. I_t is investment given by $I_t = (1 - \delta)K_t - K_{t-1}$. Using first order conditions and adjusting prices with nominal exchange rate, ε_t , gives:

$$C_{t}^{-\sigma} = \left[\beta \left\{ \frac{P_{t}}{P_{t+1}} \frac{s_{t}}{s_{t+1}} (1 + r_{t+1} - \delta) \right\} \right] C_{t+1}^{-\sigma}$$

$$N_{t} = \left[C_{t}^{-\sigma} W_{t}\right]^{\frac{1}{\gamma}} N_{t} = \left[C_{t}^{-\sigma} W_{t}\right]^{\frac{1}{\gamma}}$$
(A4)

$$\Psi_t = 1 + r_{t+1} - \delta \Psi_t = 1 + r_{t+1} - \delta$$
(A5)

$$Q_{t} = \left[\frac{P_{t+1}}{P_{t}}\tilde{\varepsilon}_{t+1}(Q_{t+1} + B_{t+1})\right]/1 + r_{t+1} - \delta$$
(A6)

The nominal exchange rate creates a link between domestic and foreign prices and output.Log linearizing and taking deviations from the steady of the consumption Euler (A3) and using the idea that $y_t = c_t$ gives

 $y_t = E_t y_{t+1} - \frac{\beta}{\sigma} [\{r_{t+1} + (1-\delta)\} - E_t (\tilde{\varepsilon}_{t+1} + \pi_{t+1})] \quad \text{canfurtherbesummarized}$

by using (A5) to obtain Equation A7.

$$y_{t} = E_{t} y_{t+1} - \frac{\beta}{\sigma} [\{i_{b,t}\} - E_{t} \tilde{\varepsilon}_{t+1} - E_{t} \pi_{t+1}]$$
(A7)

Firm

Technology is Cobb Douglas production function.

$$y_t = e^{A_t} K_t^{\alpha} N_t^{1-\alpha} \tag{A8}$$

Where:
$$A_t = \rho A_{t-1} + \varrho_t \epsilon$$
, and $\varrho_t = \varpi \varrho_{t-1} + v_t \varrho_t = \varpi \varrho_{t-1} + v_t$ is the

variance of technology shock ϵ . Profit maximization of firms is given by:

$$\Pi_{t} = Y_{t} - W_{t}N_{t} + r_{t}K_{t} \quad \dots \tag{A9}$$

first order condition:

$$W_t = (1 - \alpha) K_t^{\alpha} N_t^{-\alpha} e_t^A \tag{A10}$$

$$r_t = \alpha K_t^{\alpha - 1} N_t^{1 - \alpha} e_t^A \tag{A11}$$

First order condition of profit maximization together with household utility optimization determines equilibrium wage and real interest rate.

Price setting behaviour of firms

A fraction of firms Θ have market power that enables them to adjust price in staggered manner over time, while (1- Θ) take price as given (Calvo, 1983). It is price setting behaviour of firms that causes persistence of inflation in the economy. The Calvo price setting mechanism for firm with market power is given as:

$$\max E_{t} \sum_{t=0}^{\infty} \Theta^{t} \{ \Gamma_{t,t+\tau} \left(P_{t}^{*} Y_{t+\tau|t} - M C_{t+\tau} Y_{t+\tau|t} \right) \}$$
(A12)

In Equation A12, firms with market power maximize expected discounted profit

by choosing P_t^* , $P_t^*Y_{t+\tau|t}$ is total revenue, while $MC_{t+\tau}Y_{t+\tau|t}$ is the total cost.

$$Y_{t+\tau|t} = \left(\frac{P_t^*}{P_{t+\tau|t}}\right)^{-\Xi} C_{t+\tau|t}$$
Use the fact that
$$Y_{t+t|\tau} = C_{t+t|\tau}$$

$$\frac{\partial Y_{t+\tau|t}}{\partial P_t^*} = -\Xi \left(\frac{P_t^*}{P_{t+\tau|t}}\right)^{-\Xi-1} Y_{t+\tau|t}$$

$$= \Xi \left(\frac{P_t^*}{P_{t+k}}\right)^{-\Xi-1} Y_{t+t}$$
(A13)

Equation A14 shows that marginal revenue is declining in the increase in price P_t^* . Profit for price setting firm is equal to total revenue plus marginal revenue minus marginal cost.

$$TR_{t+\tau} + MR_{t+\tau} - MC_{t+\tau}$$

$$TR_{t+\tau} - MC_{t+\tau} = P_{t+\tau}Y_{t+\tau|t} + \frac{\partial Y_{t+\tau|t}}{\partial P_t^*} - MC_{t+\tau}\frac{\partial Y_{t+\tau|t}}{\partial P_t^*}$$
(A15)

$$= P_{t+\tau}Y_{t+\tau|t} + P_t^* \left[-\Xi \left(\frac{P_{t+\tau}}{P_t^*}\right)Y_{t+\tau} \right] - MC_{t+\tau} \left[-\Xi \left(\frac{P_{t+\tau}}{P_t^*}\right)Y_{t+\tau} \right]$$
(A16)

$$=P_{t+\tau}Y_{t+\tau|t}\left\{P_{t}^{*}+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\right\}$$
(A17)

Hence profit is maximized overtime by adjusting price as follows

$$E_{t}\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\Gamma_{t,t+\tau}\left(P_{t+\tau}Y_{t+\tau|t}\left[P_{t}^{*}+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\right]\right)\right\}$$
(A18)

Equations A16–A18 summarizes total revenue and total cost. Recall that, from

$$\Gamma_{t,t+\tau} = \beta \left[\left(\frac{C_t}{C_{t+1}} \right)^{-\sigma} \frac{P_t \varepsilon_t}{P_{t+1} \varepsilon_{t+1}} \right]$$
 the consumption Euler. Hence:
$$E_t \sum_{\tau=0}^{\infty} \Theta^{\tau} \left\{ \Gamma_{t,t+\tau} \left(P_{t+\tau} Y_{t+\tau|t} \left[P_t^* + M C_{t+\tau} \left(\frac{\Xi}{1-\Xi} \right) \right] \right) \right\} \times \frac{P_t}{P_{t+\tau}} \frac{1}{P_{t+\tau}}$$

then use
$$\frac{\mathbf{p}_t}{\mathbf{p}_{t+\tau}} = \pi_{t+\tau} \frac{\mathbf{p}_t}{\mathbf{p}_{t+\tau}} = \pi_{t+\tau}$$
 (A19)

$$E_{t}\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\Gamma_{t,t+\tau}\left(P_{t+\tau}Y_{t+\tau|t}\left[\pi_{t}^{*}+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\pi_{t+\tau}\right]\right)\right\}$$

$$E_{t}\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\left[\beta\left[\left(\frac{C_{t}}{C_{t+1}}\right)^{-\sigma}\frac{P_{t}\varepsilon_{t}}{P_{t+1}\varepsilon_{t+1}}\right]\right]\left(P_{t+\tau}Y_{t+\tau|t}\left[\pi_{t}^{*}+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\pi_{t+\tau}\right]\right)\right\}$$
(A20)

Taking logs of Equation A20 yields Equation A21.

$$E_{t}\sum_{\tau}^{\infty}\Theta^{\tau}\left\{\beta\left[y_{t}-\sigma(c_{t+\tau}-c_{t})+p_{t}\varepsilon_{t}-p_{t+\tau}\varepsilon_{t+\tau}+y_{t+\tau}+\pi_{t}^{*}-y_{t}-\sigma(c_{t+\tau}-c_{t})+mc_{t+\tau}+y_{t+\tau}\right]\right\}$$
(A21)

which can be summarized as:

$$E_{t}\sum_{\tau}^{\infty} (\Theta\beta)^{\tau} \left[\pi_{t}^{*} + p_{t}\varepsilon_{t}\right] = E_{t}\sum_{\tau}^{\infty} (\Theta\beta)^{\tau} \left[mc_{t+\tau} + p_{t+\tau}\varepsilon_{t+\tau}\right]$$
(A22)

solving infinite summation and using $\pi_t = (1 - \Theta)\pi_t^*$ and $p_t = p_t \mathcal{E}_t$

$$\pi_{\star} = \frac{(1 - \Theta)(1 - \Theta\beta)}{mc_{\star} + \beta E_{\star}(\pi_{\star,1})}$$
(A23)

Equation A23 is the Philips curve.

Where,
$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{W^{1-\alpha}}{A_t}$$
 real marginal cost¹.

The marginal costs are obtained by minimizing cost $J_t = W_t N_t + r_t K_t$ subject to a given level of output $\bar{y}_t = e^{A_t} K_t^{\alpha} N_t^{1-\alpha}$. Equation A23 is the optimal inflation that maximizes discounted revenues of firms with power to set prices.

External sector

Foreign prices influence domestic prices in open economies depending on the degree of openness and the ease of substitution between imports and import competing goods. In this regard, aggregate price index is a combination of domestic and foreign good.

$$P_{t} = \left[(1 - \Phi) (P_{H,t})^{1 - \phi - 1} + \Phi (P_{F,t})^{1 - \phi} \right]^{\frac{1}{1 - \phi}}$$
(A24)

Where: Φ is the degree of openness, \emptyset is the substitution parameter which measures the ease of substitution between domestic and foreign goods. Log linearizing the aggregate price index in Equation A24:

$$P_{t} = [(1 - \Phi)(P_{H,t}) + \Phi(P_{F,t})]$$
(A25)

Terms of trade

However, domestic price levels adjust in relation to foreign price. Hence, bilateral terms of trade (the price index of foreign goods in relation to domestic goods) between the home country *i* and trading partner country *j* is $S_{i,t} = \frac{P_{F,t}}{P_{H,t}}$; $P_{F,t}$ and $P_{H,t}$ are the foreign and the domestic price indices, respectively. In log form, $S_{i,t} = P_{F,t} - P_{H,t}$. Making $P_{F,t}$ the subject and substituting in A25.

$$P_{t} = [(1 - \Phi)(P_{H,t}) + \Phi(P_{F,t})] = P_{H,t} + \Phi s_{t}$$
(A26)

Real and nominal exchange rate can be incorporated in the model by using the purchasing power parity $P_{i,jt} = \varepsilon_{it}P_{jt}^i$ conditions. The purchasing power parity states that foreign price should be equal to domestic price in domestic currency. If the nominal exchange rate follows an Auto Regressive process of order 1 then it can be denoted as $\varepsilon_t = \psi \varepsilon_{t-1} + \omega_t \hbar_t$. The AR 1 process approximates nominal exchange rate movements in floating and managed floating exchange rate regimes. The variance of nominal exchange rate exhibit stochastic volatility given by $\omega_t = \rho_{\omega} \omega_{t-1} + v_t$ and is assumed to to be distributed as $\hbar_t \sim N(0, \omega_t)$ and $v_t \sim N(0, 1)$. Hence, foreign price index is given by , $P_{F,t} = \int_0^1 (e_{i,t} + P_{i,t}^i) \partial i = e_t + P_t^*$ where $e_{i,t}$ is the foreign bilateral nominal exchange rate and $P_{i,t}$ is the country *t* domestic price level. However, purchasing power parity condition presuppose that $P_{F,t} = \int_{-1}^{1} (e_{i,t} + P_{i,t}^i) \partial i = e_t + P_t^*$

The real exchange rate \mathbb{R} is given in Equation A27.

$$s_{i,t} = e_{i,t} + P_t^* - P_{H,t}$$
(A27)

Taking logs for Equation A27 and using linearized purchasing power parity (A26): $\mathbb{R} = \varepsilon_{i,t} \frac{P_t^{i}}{P_t} (e_{i,t} + P_{i,t}^i - P_t) \partial i \qquad (A28)$ $= e_t + P_t^* - P_t$ OPTIMAL MONETARY POLICY WITH INFLATION, OUTPUT AND ASSET PRICE VOLATILITY

$$= e_t + P_{H,t} - P_t = e_t + P_{H,t} - P_t$$
(A29)

$$=(1-\Phi)s_t \tag{A30}$$

Equation A30 is obtained by substituting expression (A26) in Equation A29. Fluctuation in foreign output influence output and asset prices of small open economies through changes in domestic demand and portfolio flows, respectively. The impact of foreign output to domestic output is approximated by assuming that world output y_t^* is an autoregressive process of order 1.

$$y_t^* = \psi y_{t-1}^* + \zeta_t$$

(A31)

Equations A4, A6, A7, A8, A9, A12, A13, A23, A26, A30, and A31 constitute a system of equations used to estimate parameters for the monetary policy response function for Ghana and Kenya.





Figure A1: Kenya: Convergence diagnostic





Notes: phi_esp is exchange rate, phi_ib is yields on bonds, phi_pi is inflation, phi_y is the output, and phi_iq is equity Source: Illustration using data from Ghana Statistical Service and GSE.

Appendix B

The estimates in Table 5[NO SUCH TABLE] do not take into account the response of the public to monetary policy decision. Hence, to establish optimal monetary policy, the dynamic responses are taken into account when setting the monetary policy rate to minimize social loss. The social loss function is a second order Taylor approximation of a separable Constant Relative Risk Aversion utility function, comprising of consumption C_t , labour N_t , and money M_t , as follows:

$$V_{t} = E_{t} \sum \beta^{t} \left[\frac{C_{t}^{1-\sigma}}{1-\sigma} - \frac{N_{t}^{1+\gamma}}{1+\gamma} + \frac{(M_{t}/P_{t})^{1-\eta}}{1-\eta} \right]$$
(B1)

First undertake second order Taylor approximation of consumption C_t , labour N_t and money M_t . Starting with consumption, C_t :

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \frac{\overline{C}_t^{1-\sigma}}{1-\sigma} + \overline{C}_t^{1-\sigma} \left[\overline{C}_t + \frac{(1-\sigma)\overline{C}_t^2}{2} \right] + \kappa^3$$
(B2)

approximate labour N_{t} :

$$\frac{N_t^{1-\gamma}}{1-\gamma} = \frac{N_t^{1-\gamma}}{1-\gamma} + \overline{N}_t^{1-\gamma} \left[\overline{N}_t + \frac{(1-\gamma)\overline{N}_t^2}{2} \right] + \kappa^3$$
(B3)

approximate financial assets M_t :

$$\frac{M_t^{1-\eta}}{1-\eta} = \frac{M_t^{1-\eta}}{1-\gamma} + \overline{M}_t^{1-\eta} \left[\overline{M}_t + \frac{(1-\eta)\overline{M}_t^2}{2} \right] + \kappa^3$$
(B4)

However, consumption C is given by C = Y-S, where S is savings. Therefore, second order Taylor approximation for output at zero steady state:

$$Y_t = \overline{Y} + \overline{Y}Y_t + \frac{1}{2}\overline{Y}Y_t^2 + \kappa^3$$
(B5)

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$$S_t = \overline{S} + \overline{S}S_t + \frac{1}{2}\overline{S}S_t^2 + \kappa^3$$
(B6)

Consumption at steady state is given by: $\overline{C}=\overline{Y}-\overline{S}$

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \frac{\overline{C}_t^{1-\sigma}}{1-\sigma} + \overline{C}_t^{1-\sigma} \left[\overline{Y}_t - \overline{S}_t + \frac{(1-\sigma)(\overline{Y}_t - \overline{S})_t^2}{2} \right] + \kappa^3$$
(B7)

Labour is used in the production function, which is summarized as: $Y_t = e^{A_t} K_t^{\alpha} N_t^{1-\alpha}$

$$N_{t} = \left(\frac{Y_{t}}{s^{A}t_{K_{t}^{\alpha}}}\right)^{1-\alpha} N_{t} = \left(\frac{Y}{A_{t}K_{t}^{\alpha}}\right)^{1-\alpha} \int_{0}^{1} \left(\frac{P_{H,t}^{i}}{P_{H,t}}\right)^{1-\xi} \partial i$$
(B8)

Let, then $n_t = (1 - \alpha)[y_t - a_t - \alpha k_t] + d_t$ in logarithm form. Let also

 $P'_{H,t} = P_{H,t}(i) - P_{H,t}$ by taking the logarithm. A second order Taylor series approximation of $\operatorname{at}\left(\frac{P^{i}_{H,t}}{P_{H,t}}\right)^{1-\xi}$ a steady state of zero after taking logarithm. Hence:

$$\left(\frac{P_{H,t}^{i}}{P_{H,t}}\right)^{1-\xi} \tag{B9}$$

$$= 1 + (1 - \xi)P'_{H,t}(i) + \frac{1}{2}(1 - \xi)\{P'_{H,t}(i)\}^2 + \kappa^2$$
(B10)

$$\int_{0}^{1} \left(\frac{P_{H,t}^{i}}{P_{H,t}} \right)^{1-\xi} \partial i = 1 - \xi E_{i} \left\{ P_{H,t}^{'}(i) \right\} + \frac{1}{2} \xi^{2} E_{i} \left\{ P_{H,t}^{'}(i) \right\}^{2} + \kappa^{3}$$
(B11)

Then taking expectation of B11 and using $E_t \left\{ P'_{H,t}(i) \right\}^2 = \frac{1}{2} (\xi) E_t (P'_{H,t}(i))^2$

$$=1+\frac{\xi}{2}\operatorname{var}_{i}\left\{P_{H,t}^{\prime}(i)\right\}+\kappa^{3}$$
(B12)

(B16)

and therefore

$$d_{t} = \int_{0}^{1} \left(\frac{P_{H,t}^{i}}{P_{H,t}}\right)^{1-\xi} \partial i = 1 + \frac{\xi}{2} \operatorname{var}_{i} \left\{P_{H,t}^{'}(i)\right\} + \kappa^{3}$$

Recall that, $P'_{H,t}(i) = P_{H,t}(i) - P_{H,t}$. This is the change in prices in the domestic economy, and hence it is inflation. Therefore, $P'_{H,t}(i) = P_{H,t}(i) - P_{H,t} = \pi_{H,t}$. $P'_{H,t}(i) = P_{H,t}(i) - P_{H,t} = \pi_{H,t}$. Then finally:

$$d_{t} = \int_{0}^{1} \left(\frac{P_{H,t}^{i}}{P_{H,t}}\right)^{1-\xi} \partial i = 1 + \frac{\xi}{2} \operatorname{var}_{i} \left\{P_{H,t}^{'}(i)\right\} + \kappa^{3} = \frac{\xi}{2} \pi^{2} + \kappa^{3}$$
(B13)

Then Equation B12 is substituted in labour supply Equation B2:

$$\frac{N_{t}^{1-\gamma}}{\overline{N}_{t}^{1-\gamma}} = \overline{N}_{t}^{1-\gamma} \left[(1-\alpha) \left[y_{t} - a_{t} - \alpha k_{t} \right] + d_{t} + \frac{(1+\gamma)((1-\alpha) \left[y_{t} - a_{t} - \alpha k_{t} \right] + d_{t} \right]}{2} + \kappa^{3}$$
(B14)

That is, marginal utility of labour equal to the share of labour in the national output. An optimal labour subsidy also enables labour suppliers to have this share. Therefore, a full labour compensation consist of its share in the national output and adjustment for inflation in d_t .

$$\frac{N_t^{1-\gamma}}{1-\gamma} = (1-\alpha) \left[d_t + \frac{(1-\gamma)((1-\alpha)y_t)^2}{2} \right] + tip + \kappa^3$$
(B15)

tip consists of α , a_t , and y_t which cannot be influenced by monetary policy. Similarly, $\overline{C}_t^{1-\sigma} = \alpha$ in Equation B3. Marginal utility of consumption is equal to share of capital in equilibrium; otherwise, household will adjust share of output consumed and saved

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \alpha \left[y_t + \frac{(1-\sigma)^2 y_t^2}{2} \right] + tip + \kappa^3$$

Equation B16 uses Equation B5 and Equation B6 in substituting for $\overline{Y_t} - \overline{S_t}$ in Equation B7.

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Second order Taylor approximation for equity and bond price

Financial assets consist of money, bonds, and equity from the budget constraint:

$$\Psi = M_t + B_t \int_0^1 \left(\frac{F_{H,t}(i)}{F_{H,t}}\right)^{-e} \partial i + Q_t \int_0^1 \left(\frac{Z_{G,t}(i)}{Z_{G,t}}\right)^{-\varsigma} \partial i$$
(B17)

Following a similar procedure as inflation, equity and bond price approximations are:

$$B_{t} \int_{0}^{1} \left(\frac{F_{H,t}(i)}{F_{H,t}} \right) \quad \partial i = 1 + \frac{e}{2} \operatorname{var}_{i} \{ F_{H,t}'(i) \} + \kappa^{2}$$
(B18)

$$Q_{t} \int_{0}^{1} \left(\frac{Z_{G,t}(i)}{Z_{G,t}} \right)^{-\varsigma} \partial i = 1 + \frac{\varsigma}{2} \operatorname{var}_{i} \{ Z_{G,t}'(i) \} + \kappa^{2}$$
(B19)

The financial assets are summarized in M_t in the utility Equation B1. Approximating equity and bonds in M_t :

$$\frac{M_t^{2\eta}}{1-\eta} = \overline{M}_t^{1-\eta} \left[(\overline{B}_t + \widetilde{B}_t + \overline{Q}_t + \widetilde{Q}_t) + \frac{(1-\eta)(\overline{B}_t + \widetilde{B}_t + \overline{Q}_t + \overline{Q}_t)^2}{2} \right] + \kappa^2$$
(B20)

$$\frac{M_t^{1-\eta}}{1-\eta} = \alpha \left[(\widetilde{B}_t + \widetilde{Q}_t) + \frac{(1-\eta)(\widetilde{B}_t + \widetilde{Q}_t)^2}{2} \right] + tip + \kappa^2$$

$$\widetilde{B}_t$$
(B21)

and \widetilde{Q}_t in Equation B20 can be substituted for expressions in (B20) and (B21), respectively. Meanwhile, Equation B21 is summarized by lumping together terms of raised to power greater than two in *tip* because they are not influenced by monetary policy. Substituting equations B15, B16, and B21, all the terms in the utility function:

$$V_{t} = E_{t} \sum_{t=0}^{\infty} \beta^{t} \left[\alpha \left[\frac{y_{t}}{2} + \frac{(1-\sigma)^{2} y_{t}^{2}}{2} \right] - (1-\alpha) \left[d_{t} + \frac{(1+\gamma)((1-\alpha)y_{t})^{2}}{2} \right] + \alpha \left[(\widetilde{B}_{t} + \widetilde{Q}_{t}) + \frac{(1-\eta)(\widetilde{B}_{t} + \widetilde{Q}_{t})^{2}}{2} \right] + tip + \kappa^{3} \right]$$

(B22)

$$=E_{t}\sum_{t=0}^{\infty}\beta^{t}\left[\alpha\left[\frac{(1-\sigma)}{2}y_{t}^{2}\right]-\frac{(1-\alpha)\pi_{H,t}^{2}}{2}-\frac{(1-\alpha)(1+\gamma)(1-\alpha)^{2}y_{t}^{2}}{2}+\frac{\alpha e}{2}\widetilde{B}_{t}^{2}+\frac{\alpha \zeta}{2}\widetilde{Q}_{t}^{2}+tip+\kappa^{3}\right]$$
(B23)

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$$=E_{t}\sum_{t=0}^{\infty}\beta^{t}\left[\left[\frac{\alpha(1-\sigma)-(1-\alpha)(1+\gamma)(1-\alpha)^{2}}{2}\right]y_{t}^{2}+\frac{(1-\alpha)\pi_{H,t}^{2}}{2}+\frac{\alpha e}{2}\widetilde{B}_{t}^{2}+\frac{\alpha \varsigma}{2}\widetilde{Q}_{t}^{2}+tip+\kappa^{3}\right]$$
(B24)

Let $\gamma_{\pi} = \frac{(1-\alpha)}{2}$; $\gamma_{y} = \frac{\alpha(1-\alpha) - (1-\alpha)(1+\gamma)(1-\alpha)^{2}}{2}$; $\gamma_{\varrho} = \frac{\alpha e}{2}$; $\gamma_{B} = \frac{\alpha \zeta}{2}$ and dropping *tip* and κ^{3} Then substituting in Equation B24:

$$L_{t} = E_{t} \sum_{t=0}^{\infty} \beta^{t} \left[\gamma_{y} y_{t}^{2} + \gamma_{\pi} \pi_{H,t}^{2} + \gamma_{B} \widetilde{B}_{t}^{2} + \gamma_{Q} \widetilde{Q}_{t}^{2} \right]$$
(B25)

Equation B25 is the social welfare function, which describes social preference for inflation $\pi_{H,t}$, output \mathcal{Y}_t , equity Q_t , and bond prices B_t . Therefore, optimality of monetary policy can be evaluated by using the loss value. A low value indicates that the social loss from inflation $\pi_{H,t}$, output \mathcal{Y}_t , equity Q_t , and bond prices B_t deviation from the socially desired level is minimized. The response of the public to monetary policy decision is informed by the actual level of inflation $\pi_{H,t}$, output \mathcal{Y}_t , equity Q_t , and bond prices B_t deviation from the social prices B_t relative to the desired level. The social problem solved by monetary policy involves minimizing Equation B25, subject to the evolution of output gap, inflation, and asset prices:

$$L_{t}_{Minimise\{y_{t},\pi_{H,t},\widetilde{Q}_{t},\widetilde{B}_{t}\}} = E_{t} \sum_{t=0}^{\infty} \beta^{t} \left[\gamma_{y} y_{t}^{2} + \gamma_{\pi} \pi_{H,t}^{2} + \gamma_{B} \widetilde{B}_{t}^{2} + \gamma_{Q} \widetilde{Q}_{t}^{2} \right]$$
(B26)

$$y_{t} = E_{t} y_{t+1} - \frac{\beta}{\sigma} \left[\left\{ i_{b,t} \right\} - E_{t} \tilde{\varepsilon}_{t+1} - E_{t} \pi_{t+1} \right]$$
(B27)

$$Q_{t} = \left[\frac{P_{t+1}}{P_{t}}\tilde{\varepsilon}_{t+1}(Q_{t+1} + B_{t+1})\right]/1 + r_{t+1} - \delta$$
(B28)

$$\pi_t = \frac{(1-\Theta)(1-\Theta\beta)}{\Theta} mc_t + \beta E_t(\pi_{t-1})$$
(B29)

$$\Psi_{t+1} = [1 + r_{t+1} - \delta] \tag{B30}$$

Equation B26 is the state equation, and L is the variable to be minimized. γ_{π} , γ_{y} , γ_{Q} , γ_{B} are contributions of inflation, income, equity price, and debt to social loss. They are also weights of respective variable in the social loss function. Equations B27–B30 are decision equations which influence the state equation. The decision variable is *i* interest rate. That is, *i* is changed to ensure that *L* is minimized subject to equations B27–B30.



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