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ECON 641: MONETARY ECONOMICS I

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ECON 641: MONETARY ECONOMICS I

Topic 1: Micro and Macro Perspectives of Monetary Economics

Contents

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1. MICRO AND MACRO PERSPECTIVES OF MONETARY ECONOMICS

1.1 Money in the Utility Functions

The Neo-classical growth model provides the basic framework for much of modern macroeconomics. The Solow growth model has three key ingredients:

a) A production function allowing for smooth substitutability between labor and capital in the production of output.

b) A capital accumulation process in which a fixed fraction of output is devoted to investment each period, and

c) A labor supply process in which the quantity of labor input grows at an exogenously given rate. Solow showed that such an economy would converge to a steady-state growth path along which output, capital stock, and effective supply of labor all grow at the same rate.

When the assumption of a fixed savings rate is replaced by a model of forward-looking households choosing savings and labor supply to maximize lifetime utility, the Solow model becomes the foundation for dynamic stochastic models of the business cycles.

The neoclassical growth model is a model of a non- monetary economy, and while goods are exchanged and transactions must be taking place, there is no medium of exchange, i.e no money, to facilitate these transactions. Nor is there any asset, like money, that has a zero nominal rate of return and is therefore dominated in rate of return by other interest-bearing assets. To employ the neoclassical framework to analyze monetary issues, a role for money must be specified so that agents will wish to hold positive quantity of money. A positive demand for money is necessary, if in equilibrium, money is to have positive value.

A fundamental question in monetary economics is the following:

1. How should we model the demand for money?

2. How do real economies differ from Arrow-Debreu economies in ways that give rise to a positive value for money? The Arrow–Debreu model applies to economies with maximally complete markets, in which there exists a market for every time period and forward prices for every commodity at all time periods and in all places.

The Arrow–Debreu model is one of the most general models of competitive economy and is a crucial part of general equilibrium theory, as it can be used to prove the existence of general equilibrium (or Walrarian equilibrium) of an economy.

Three general approaches to incorporating money into general equilibrium models have been developed.

a) Those that assume that money yields direct utility by incorporating money balances directly into the utility functions of the agents of the model (Sidrauski, 1967)

b) Those that impose transactions costs that give rise to a demand for money, by making assets exchanges costly (Baumol, 1952, Tobin, 1956), requiring that money be used for certain types of transactions.

c) Those that treat money like other asset used to transfer resources inter-temporally (Samuelson, 1958).

The Basic MIU Model:

To develop the model, we will ignore uncertainty and labor-leisure choice, focusing on the implications of the model for money demand, the value of money, and the cost of inflation.

Suppose the utility function of the representative household takes the form:

$$U_t = U(C_t, Z_t) \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where Z_t is the flow of services yielded by money holdings and C_t is time 't' per capita consumption. It is assumed that $\partial U/\partial C > 0$ and $\partial U/\partial Z > 0$. The demand for money services will always be positive if we assume that limit as Z tends to zero, $U_z(C, Z) = \infty$, for all C where $U_z = \partial U/\partial z(C,Z)$.

What constitutes Z_t? Assuming rational economic agent, then what enters the utility function is not the number of dollars (or Shillings) that the individual holds, but the command over goods that are represented by those dollar holdings, or some measure of transaction services, expressed in terms of goods, that money yields. In other words,

$$Z_t = M_t/p_tN_t = m_t \text{ (real per capita money holding, N = population size)} \ldots\ldots \quad (2)$$

To ensure the monetary equilibrium exists, it is often assumed that marginal utility of money eventually becomes negative for sufficiently high money balances.

The representative household is viewed as choosing lifetime paths for consumption and real money balances subject to budget constraints to be specified below with total utility given by:

$$W = \sum_{t=0}^{\infty} \beta_t U(C_t, m_t) \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

Where $0 < \beta < 1$ is a subjective rate of discount. To complete the specification of the model, we assume that household can hold money, that bonds pay a nominal interest rate i_t, and physical capital. Physical capital produces output according to the standard neoclassical production function. Given its assets and any other net transfer received from the government, (τ_t), the household allocates its resources between consumption (C), gross investment in physical capital (K), and gross accumulation of real money balances (M/p) and bonds (B).

If the rate of depreciation of physical capital is δ, the aggregate economy – wide budget constraint of the household sector takes the form;

$$Y_t + \tau_t N_t + (1-\delta)K_{t-1} + [(1+i_{t-1})B_{t-1}]/p_t + M_{t-1}/p_t = C_t + K_t + M_t/p_t + B_t/p_t \quad \ldots\ldots (4)$$

Where Y = aggregate output

C = Consumption

B = Bonds

M/p = Real money balances

K_{t-1} = aggregate stock of capital at the start of period t

$\tau_t N_t$ = the aggregate real value of any lump- sum transfers (taxes if negative).

The timing implicit in this specification of the MIU model assuming that it is the household's real money holdings at the end of the period, M_t/p_t after having purchased consumption goods that yield utility.

Let $Y_t = f(K_{t-1}, N_t)$

Assuming that production function is linearly homogenous with constant returns to scale, output per capita at time, t, will be a function of per capital stock. i.e.

$$Y_t = f\,[K_{t-1}/(1+n)] \quad \ldots\ldots (5)$$

Where n = population growth rate (assumed constant). Dividing both sides of equation (4) by N_t, the per capita version becomes:

$$W_t \equiv f[K_{t-1}/(1+n)]+\tau_t+[(1-\delta)/(1+n)]K_{t-1}+[(1+i_{t-1})b_{t-1}+m_{t-1}]/(1+\pi_t)(1+n)=c_t+k_t+m_t+b_t \ldots\ldots (6)$$

Where: $\pi_t =$ rate of inflation, $b_t = B_t/p_tN_t$ and $m_t = M_t/p_tN_t$.

The household problem is to choose paths for c_t , k_t , b_t and m_t to max. equation (3), substitute it into equation (6), i.e.,

$$\text{Max } W = \sum_{t=0}^{\infty} \beta_t U(C_t, m_t)$$

Subject to,

$$w_t = c_t + k_t + m_t + b_t \quad \ldots\ldots (7)$$

This is a problem in dynamic optimization and it is convenient to formulate the problem in terms of the value function.

The value function defined as the present discounted value of utility if the household optimally chooses consumption, capital holdings, bond holdings and money balances, is given by;

$V(w_t) = \max \{U(c_t, m_t) + \beta v(w_{t+1})\}$ ………………………………………………. (7)

Where the maximization is subject to the budget constraint (4) and

$W_{t+1} = f(k_t)/(1+n) + \tau_{t+1} + [(1-\delta)/(1+n)]k_t + [(1+i_t)b_t + m_t]/(1+л_{t+1})(1+n)$ ……… (8)

Using equation (6) to express $k_t = w_t - c_t - m_t - b_t$ and making use of the definition of $w_{t+1,}$ equation (7) can be written as;

$V(w_t) = \max \{U(c_t, m_t) + \beta v[f(w_t - c_t - m_t - b_t)]/(1+n) + \tau_{t+1} + [(1-\delta)/(1+n)](w_t - c_t - m_t - b_t) + [(1+i_t)b_t + m_{t-1}]/(1+л_{t+1})(1+n)\}$

First Order Conditions:

For c_t: $u_c(c_t, m_t) - \beta/(1+n)[f_k(k) + 1 - \delta)Vw(w_{t+1})] = 0$ ……………………………… (9)

For b_t: $[(1 + i_{t)}/(1+л_{t+1})(1+n)] - \{[f_k(k_t) + 1-\delta]/(1+n)\} = 0$ …………………………….. (10)

For m_t: $u_m(c_t, m_t) - \beta\{[f_k(k_t) + 1-\delta]/(1+n)\}Vw(w_{t+1}) + [\beta Vw(w_{t+1})/(1+л_{t+1})(1+n)] = 0$ … (11)

Together with the transversality condition,

$\text{Limit}_{t\to\infty}\beta^t \lambda_t x_t = 0$, for x= k, b, m where λ_t is the marginal utility of period t consumption. The enveloped theorem implies; $\lambda_t = u_c(c_t, m_t) = Vw(w_t)$ ……………………………… (12)

Since initial resources w_t must be divided between consumption, capital, bond and money balances, each use must yield the same marginal benefit as an optimal allocation.

Using equation (9) and (12) equation (11) can be written as;

$u_m(c_t, m_t) + [\beta u_c(c_{t+1}, w_{t+1})/(1+л_{t+1})(1+n)] = u_c(c_t, m_t)$ ………………………………. (13)

Which states that the marginal benefit of adding to money holdings at time, t, must equal the marginal utility of consumption at time, t. The marginal benefit of additional money holdings has two components. First, money directly yields utility u_m. Second, real money balances at time, t, add $1/(1+л_{t+1})(1+n)$ to real per capita resources at time t+1. This addition to w_{t+1} is worth $V_w(w_{t+1})$ at time t+1, or $\beta v_w(w_{t+1})$ at time t. Thus, the total marginal benefit of money at time t is $u_m(c_t, m_t) + [\beta v_w(w_{t+1})/(1+л_{t+1})(1+n)]$.

From equation (9), (10) and (13),

$u_m(c_t, m_t)/u_c(c_t, m_t) = 1 - [1/(1+Л_{t+1})(1+n)][\beta u_c(c_{t+1}, m_{t+1})/u_c(c_t, m_t)]$

$= 1 - [1/(1+л_{t+1})(1+r_t)] = i_t/(1+i_t)$ ……………………………………………. (14)

Where: $1+r_t \equiv f_k(k_t) + 1-\delta$ is the real return on capital.

r_t is the relative price of real money balances in terms of the consumption good. The marginal rate of substitution between money and consumption is set equal to the price, or opportunity cost of holding money. The opportunity cost of holding money is directly related to the nominal rate of interest. The household could held one unit less of money, purchasing instead a bond yielding a nominal return of i; the real value of this payment is i/(1+л), and since it is received in period t+1, its present value is i/(1+r)(1+л) = 1/(1+i) .

Because money is assumed to pay no rate of interest, the opportunity cost of holding money is affected by both the real return on capital and by the rate of inflation. If the price level is constant (so л = 0), then the forgone earnings from holding money rather than capital are determined by the real return to capital. If the price level is rising (л>0), this causes the real value of money to decline, adding to the opportunity cost to holding money.

Equation (9) for capital holdings has a similar interpretation; the net marginal return from holding additional capital, $\{\beta[f_k(k_t) + (1-\delta)]V_w(w_{t+1})\}/(1+n)$, must equal the marginal utility of consumption.
Equation (10) links the additional return on bonds, inflation, and the real return on capital. It can be written as;

$$1+i_t = [1+ f_k(k_t) - \delta](1+ л_{t+1}) = (1+r_t)(1+ л_{t+1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

This relationship between real and nominal rates of interest is called the Fisher relationship after Irving Fischer (1896). It expresses the gross nominal rate of interest as equal to the gross real return on capital times 1 plus the expected rate of inflation.
Note that:
(1+x)(1+y) ≈ 1+x+y when x and y are small, then equation (15) is often written as:

$$i_t = r_t + л_{t+1}$$

Equation (9) to (11), together with the budget constraint (6), characterizes the household’s choice of consumption, money, bond, and capital holdings at each point in time. Equilibrium also requires that the nominal demand for money equals the nominal supply of money (assumed to be exogenous).

See specific examples;

1. Cobb-Douglas production function
2. CES production function.

1.2 Shopping-Time Models

Let us now turn our attention to a situation where money enters the utility function indirectly. This is known as the Shopping-Time Model. It is sometimes asserted that money does not directly yield consumption services to the individual, but that its use saves on the time spent in making payments. This first part of this assertion implies that the first two axioms of preferences in the preceding subsection are not applied to real balances but only to commodities and leisure. A model that leaves real balances out of the direct utility function but embodies their usage for facilitating purchases and sales of commodities is briefly specified in this subsection.

For this model, assume that only consumer goods and leisure directly yield utility. Hence, the one-period utility function U(.) is:

The one-period utility function U(.) is: U(.) = U(c,L) (1)

Where: c = consumption L = leisure.

Assume that Uc,ULS > 0, Ucc,ULL > 0.

Note that consumption requires purchases of consumer goods, which necessitate time for shopping. This shopping time can be divided into two components, one being the selection of the commodity to be purchased and the other that of making the payment acceptable to the seller.

The former is often enjoyable to most people and can be treated as an aspect of the commodity bought, or as a use of leisure, or ignored as a simplification device for our further analysis. The second component is an aspect of the payments system. If the buyer does not have enough of the medium of payments to pay for the purchase, he has to devote time to getting it, say, from a bank, or to find a seller who will be willing to accept the payment in the commodity or labor services that the seller can provide, where the latter is the time taken by bartering.

These two clearly take time. In a monetary economy, over all his purchases, the buyer needs a certain amount of money to buy all the goods and services that he wishes to purchase. He can hold enough or only some proportion of this amount. If he holds less than 100 percent of the amount needed, he will have to devote part of his time to effect the remaining payment by devoting some time to the payments process. The amount of time needed for this purpose will be positively related to the shortfall in his money holdings. The time used for this purpose is a nuisance, would have negative marginal utility and can be labeled as "payments time" – that is, the time needed to effect the payments for the commodities bought. It is also often labeled as "shopping time" or "transactions time."

Leisure equals the time remaining in the day after deducting the time spent on a job and the payments time. Hence, L = h0 - n - nT (2)

Where:

h0 = maximum available time for leisure, work and transactions

n = time spent working

nT = payments time, i.e. time spent in making payments in a form acceptable to the seller

The payments and financial environment are assumed to be such that the "payments/ transactions time function" is:

$$nT = nT(mh, c) \quad (3)$$

Where: $\partial nT/\partial c > 0$ and $\partial nT/\partial mh \leq 0$.

From (2) and (3), $\partial U/\partial nT = (\partial U/\partial L)(\partial L/\partial nT) < 0$.

That is, an increase in payments time decreases leisure and therefore decreases utility. But, since an increase in the amount held and utilized of real balances decreases payments time;

$\partial U/\partial mh = (\partial U/\partial nT)(\partial nT/\partial mh) > 0$.

A proportional form of the payments time function is:

$$nT/c = \varphi(mh/c) \quad (4)$$

Where: $-\infty < \varphi \leq 0$, with $\varphi\_$ as the first-order derivative of φ with respect to mh/c. Satiation in real balances occurs as $\varphi\_ \rightarrow 0$. (3) implies that $\partial\varphi/\partial mh \leq 0$. Incorporating this payments time function into the utility function above (1), we have:

$$U(.) = U(c, h0 - n - c\varphi(m/c)) \quad (5)$$

Equation (5) can be rewritten as the indirect utility function: V(.) = V(c,n,mh)

According to Handa (2009), the generic form and properties of the indirect utility function, which has real balances as a variable, are similar to those of the direct one used earlier in this. Therefore, economists who prefer its payments time justification for putting money in the utility function substitute this justification for the one given earlier for the direct MIUF, which was simply that money is in the utility function because the individual prefers more of it to less, ceteris paribus, in the environment of a monetary economy. Both justifications are acceptable. However, given the similarity of the direct and the indirect utility functions, and the relative simplicity of using the former, we revert for convenience to the direct utility function.

1.3 Overlapping Generation Model

Overlapping generations (OLG) models of money, which was first introduced by Samuelson (1958), and later with major extensions by Wallace (1980, 1981) have been proposed by some economists as an alternative to the money in the MIUF). However, other economists do not consider the OLG models of money in their standard form to be valid or useful for modeling the actual role of money in the economy.

Basing our presentation on Handa (2009) and the work of Wallace (1980, 1981), we assume the standard version of the OLG model assumes that the individuals in the economy live for two periods only – or for two life-stages, “young” and “old,” with each life-stage lasting one period – and that in each period the economy has two generations of individuals. One of these is the old generation of individuals who were born in the preceding period and the other is the young generation born at the beginning of the current period. The old of one generation and the young of the next one overlap in every period, so that the name given to the models using this framework is the overlapping generations models.

The OLG framework is a substitute for a timeless or an infinite one, with the representative agent having an infinite horizon. It does not by itself provide a model but has to be combined with other assumptions in order to yield a meaningful model.

The essential assumptions and implications of the OLG models with fiat money

The assumptions of the standard OLG models with money are:

- Defining bonds as interest-bearing financial assets that can be used to convey purchasing power from the present to the future, there are no bonds in the model.
- Fiat money is preferable to commodities – and any other assets – as the medium for carrying forward saving to the following period.
- There is net (positive) saving in the first lifestage.
- Future periods will not renounce the use of fiat money or pursue policies such that fiat money will become valueless.
- The OLG model’s economy has an infinite horizon, even though the individuals in it have a finite (two-period) horizon.

Given these assumptions, the OLG models of fiat money explore the value of money for various growth rates of money versus commodities, growth of population, open market operations, etc. Among the attractive features claimed for OLG models is that, along certain paths, they establish a positive value for an intrinsically worthless fiat money10 which is not required by law to be convertible into commodities, and that time and the distinctiveness of the earning pattern over a lifetime are incorporated in an “essential” manner.11 Further, they allow for economic agents who are identical at birth – thus permitting the study of stationary states – while allowing for a degree of heterogeneity among the economic agents alive at any time in the economy, and also allow – indeed require – the economy to continue indefinitely into the future.

As pointed out already, OLG models with money generate a zero value of money in the current period if the value of money is expected to be zero in some future period. This is a characteristic of bootstrap or bubble paths, which are paths along which the values of the variables depend upon expected values, even if arbitrary ones, in the future and change if the latter change. The numerous equilibria of this kind are among the tenuous kind, meaning by this that they are not based on the fundamentals of the system. However, the usual focus of OLG models is not on such bootstrap or bubble paths. Rather, their implications are normally analyzed only for the stationary states of the economy, with expectations assumed to be identical with the stationary values or with those implied by the rational expectations hypothesis (REH)

The Basic OLG Model

In the standard version of the OLG framework, individuals live for two periods – that is, go

through two life-stages – only. They are often labeled “young” in their first lifestage and “old” in their second lifestage. This book uses the superscripts y and o to indicate the individual’s respective life-stages.

For the economy, the periods are t +i, i = 0,1,2, ... Period t is the initial period of the analysis and its old generation is called the “initial old,” whose members were born in period t −1. Generations born in periods 0,1,2, . . ., will be called the “future generations” and its members will be referred to merely as “individuals.” The OLG model starts by endowing the initial old with the initial stock of money. Further, for the basic OLG model of this chapter, it is assumed that any increase in the money stock in any period is gratuitously given as a lump-sum transfer to the old in that period. The next chapter deviates from this assumption to examine the case where the seigniorage from money creation is used to buy up commodities that are then destroyed, resulting in a net decrease in the commodities left for consumption in the economy.

The number of individuals born in period t is Nt. In the early parts of the analysis of this section, this number is assumed to be constant at N over time. Under this assumption, in each period t, the population of 2N individuals consists of N young individuals and N old individuals.

Each individual is assumed to be given a commodity endowment of Wy in the young life-stage and wo in the old life-stage. Wy and wo are in units of the single consumption good, assumed in the basic model to be non-storable (perishable). Some of the versions of the OLG models assume that wo is zero, but such an assumption is not essential to the OLG framework.

However, if fiat money is to have value, it is essential to assume that the optimal level of consumption in old age will exceed wo. This is usually guaranteed by an assumption that consumption will be the same in each life-stage and that wo < wy, so that the individual must have while young to provide for extra consumption in the second period.

Intertemporal Budget Constraint of the Young

In the young life-stage, the representative individual can either consume c y or hold money m out of his endowments of commodities. His budget constraint for the first/young life-stage is:

$$p_t c^{y}_t + m^{y}_t = p_t w^{y}_t \quad c^{y}_t < w^{y}_t$$

At the beginning of period t + 1, the individual has the carryover money balances of mt (which do not pay interest) and receives gratuitously the (real) endowment of commodities Wot +1, so that his second/old life-stage constraint is:

$$p_{t+1} c^{o}_{t+1} = p_{t+1} w^{o}_{t+1} + m^{o}_{t+1}$$

Where the money balances purchased when young, myt , become the inheritance of the old as mot+1, so that myt = mot+1 = mt .

It is noteworthy that there is no explicit interest rate in this model since the commodity is perishable and there are no interest (or coupon) paying assets in the model. The only asset is money, which does not pay interest, so that the interest rate does not enter (2). Note also that the individuals are assumed to have perfect foresight over the future values of the variables. From (2),

$$m^{o}_t = p_{t+1} c^{o}_{t+1} - p_{t+1} w^{o}_{t+1}$$

Noting that mot+1 = myt , substitution of (2') in (1) gives the individual's lifetime budget

constraint as:

$$p_t c^{y}_t + p_{t+1} c^{o}_{t+1} = p_t w^{y}_t + p_{t+1} w^{o}_{t+1}$$

Define the individual's real lifetime wealth W_t as:

$$W_t = w^{y}_t + (p_{t+1}/p_t) w^{o}_{t+1}$$

The symbols used so far and their definitions are:

C^{y}_{t} = consumption of the young in period t
C^{0}_{t}= consumption of the old in period t
P_t =price of goods in period t
W^{y}_{t}=exogenous real income of the young in period t
W^{0}_{t}=exogenous real income of the old in period t
W_t =lifetime wealth in period t\
$N_{t-1} + N_t$=number of persons born in period t

m_t^y=per capita demand for nominal balances by the young in period t
W_t^0=money endowment of each old individual in period t
M_t=total amount of fiat money in period t(=$N_t^0 m_t^0$).

Since $C_t^y < W_t^y$ by assumption, the young want to transfer commodities to themselves in the future, but the non-storable commodity assumption of the model prevents them doing so directly – as it were, through barter (via storage) between themselves when young and when old. Further, the auctioneer and other costless clearing mechanisms of the general Walrasian equilibrium models are excluded from the OLG models. So are state-enforced compulsory exchanges between generations, as through a government pension or social security system. Similarly excluded are private intergenerational mechanisms for transfers of commodities between generations through a private pension plan or an extended family system. The OLG models only allow the transfer of commodities over generations through trade, with the intermediation of money.

Utility Maximization by the Young

The individual born in period t has an intertemporal utility function:

U(c yt , cot+1)

Where U(.) is assumed to be an ordinal utility function with continuous first- and second-order partial derivatives.

The young maximize this intertemporal/lifetime utility function subject to the lifetime budget constraint (3). That is, the young's optimization problem is:

$$\text{Maximize} \quad U(c^y{}_t, c^0{}_{t+1}) \qquad (4)$$
$$\text{subject to:} \quad p_t c^y{}_t + p_{t+1} c^0{}_{t+1} = p_t w^y{}_t + p_{t+1} w^0{}_{t+1} \qquad (3)$$

Implying the optimal consumption amounts cyt , cot+1 as:

$$c^y{}_t = c^y{}_t(p_{t+1}/p_t, w^y{}_t, w^o{}_{t+1})$$
$$c^o{}_{t+1} = c^o{}_{t+1}(p_{t+1}/p_t, w^y{}_t, w^o{}_{t+1})$$

By assumption, with $w^y{}_t > w^o{}_{t+1}$,

$$c^y{}_t < w^y{}_t$$
$$c^o{}_{t+1} > w^o{}_{t+1}$$

The net dissaving in the old life-stage is accomplished by spending the money balances carried over from the young life-stage. Optimal saving syt in period t is given by:

$$s^{y}_{t} = w^{y}_{t} - c^{y}_{t}$$
$$= s^{y}_{t}(p_{t+1}/p_{t}, w^{y}_{t}, w^{o}_{t+1})$$

The demand for money, identical with that for nominal saving, is:

$$m^{y}_{t} = p_{t}s^{y}_{t} = p_{t}s^{y}_{t}(p_{t+1}/p_{t}, w^{y}_{t}, w^{o}_{t+1})$$

Intuitively, in period t, the young individual receives more of the consumption good than he wants to consume but cannot store the excess since the consumption good is perishable. He sells it to the initial old for fiat money, provided that he expects to be able to exchange his fiat money holdings for the consumption good in period t +1.

Utility Maximization by the Initial Old

From the perspective of the initial old in the initial period t, they receive some of the consumption good. Further, while they received fiat money, its utility in consumption is zero so that they are willing to exchange it for some amount of the consumption good. Formally, the utility function and budget constraint, respectively, of the initial old are:

$$U^{o}_{t} = U(c^{o}_{t})$$
$$p_{t}c^{o}_{t} = p_{t}w^{o}_{t} + m^{o}_{t}$$

Each member of the initial old maximizes his utility by maximizing cot, which implies that he will try to trade mot for the maximum amount that he can get of the consumption good.

Macroeconomic Analysis: The Price Level and the Value of Money

There are only two goods, the commodity and money, in this OLG model, so that the macroeconomic analysis has to take account of only the markets for money and the commodity. Further, by Walras's law, equilibrium in one of these markets ensures equilibrium in the other one, so that we need to present the analysis of one market only. We choose to focus explicitly on the money market for further analysis.

For the economy in period t, the aggregate demand for nominal balances Mdt equals the nominal value of the commodities the young want to sell, so that it is given by:

$$M^{d}_{t} = N_t[p_t(w^{y}_{t} - c^{y}_{t})] \quad (10)$$

The money supply M_t in the economy is given by the money balances held by the old (born in $t-1$ with their number as N_{t-1}). The old want to trade it for commodities. This amount equals:

$$M_t = N_{t-1}[m^{o}_{t}] \quad (11)$$

so that money market clearance, with money demand equal to money supply, implies that:

$$N_t[p_t(w^{y}_{t} - c^{y}_{t})] = M_t \quad (12)$$

$$p_t = M_t/[N_t(w^{y}_{t} - c^{y}_{t})] \quad (13)$$

From (5), c^{y}_{t} on the right side of (12) depends on p_{t+1}/p_t, w^{y}_{t} and w^{o}_{t+1}, so that:

$$p_t = M_t/[N_t(w^{y}_{t} - c^{y}_{t}(p_{t+1}/p_t, w^{y}_{t}, w^{o}_{t+1}))] \quad (13')$$

Hence, *ceteris paribus*, the price level p_t varies proportionately with the money supply M_t, which is a quantity theory result. Further, note that p_t depends on the intertemporal price ratio p_{t+1}/p_t.

From (13), the value v_t per unit of money, which is equal to $1/p_t$, is given by:

$$\begin{aligned} v_t &= [(w^{y}_{t} - c^{y}_{t})]N_t/(N_{t-1}m^{o}_{t}) \\ &= [(w^{y}_{t} - c^{y}_{t})N_t]/M_t \end{aligned} \quad (14)$$

where $c^{y}_{t} = c^{y}_{t}(p_{t+1}/p_t,\ w^{y}_{t},\ w^{o}_{t+1})$. Hence, the value of money is positive and changes inversely with the money supply. It also varies proportionately with aggregate saving $[(w^{y}_{t} - c^{y}_{t})N_t]$.

1.4 Cash-in-Advance Models (Clower Constraint)

The cash-in-advance constraint, also known as the Clower constraint after American economist Robert W. Clower (1967). The basic cash in advance model is due to Lucas. Every period a consumer has to choose (a) their consumption (denoted c) (b) their money balances (denoted m) and (c) their savings (denoted a, assets). However, all consumption goods have to be paid for by cash so there is a constraint the consumer faces, $P_tC_t \leq m_t$. Assets deposited in the bank earn an interest rate $R > 0$ but no interest is earned on assets held in the form of money. Instead, money earns a rate of return equal to P_{t-1}/P_t, so if there is inflation money earns a negative return (it loses value).

Consumers choose their consumption, assets and money balances once they observe the state of the world (i.e. after seeing what today's money supply growth is, what the value of the current productivity shock is, etc.). Because consumers earn interest on deposits but not on money they will always prefer to keep assets on deposit. Therefore, they will hold only just enough cash to finance their consumption, e.g. PtCt = mt. This has a rather unfortunate consequence that the velocity of money is constant. The velocity of money (V) is defined by the identity MV = PY, where M is the money supply, P is the price level and Y is the volume of transactions in the economy. Assuming no capital, the volume of transactions in this economy is just c, and because m = PC it must be that the velocity of money is always equal to 1. In reality, the velocity of money shows considerable variation and depends in particular on the interest rate. These are features which the basic cash in advance (CIA) model cannot account for.

Svensson (1985) proposes a simple amendment to Lucas' basic model. Like Lucas' article, Svensson's main concern is how to price assets when you have a cash in advance constraint. Svensson assumes that consumers have to choose how much cash to hold before they know the current state of the world (i.e., they are ignorant of the current money supply or productivity shock). As a result of this uncertainty the velocity of money is no longer constant. Agents will usually choose to hold m > Pc for precautionary reasons.

In a very good state of the world, agents know they would like their consumption to be high and they can only achieve this if they have high money balances. Therefore, agents tend to hold more money than they otherwise would need as a precaution in case they find themselves wanting to consume large amounts in a surprisingly good state of the world. The greater the uncertainty facing the consumer (e.g., the higher the probability of wanting to spend a lot on average) the larger these precautionary balances. However, the higher is the interest rate the lower the level of precautionary balances held by the consumer. Consumers have to trade the benefits of higher money balances (increased insurance against a good state of the world) against the costs (loss of interest). As a result, the velocity of money becomes time-varying and depends on the interest rate.

Cash-Credit Models

Another version of the CIA model is the so-called cash-credit model of Lucas and Stokey (1987). In this model agents gain utility from two goods, c1 and c2, where c1 can only be purchased using cash but c2 can be purchased on credit. The timing of the model is as follows. Agents observe the state of the world, decide on c1 and c2 and m, they then go and purchase cash goods paying for them with their money balances and also purchase credit goods, and then at the end of the period all credit bills are settled. This is another way of making the velocity of money variable. In this model, agents get utility from two goods, but on one good they have to pay cash and so lose R on any assets held in the form of cash. Therefore, when the interest rate is high, they will tend to lower c1 and increase c2 to compensate, because they consume less of the cash good, they also hold fewer money. Therefore, the velocity of money ((c1 +c2)/m) varies positively with the interest rate - the higher the interest rate, the lower are money balances, and the harder money has to work.

Let us now consider the following problem. Suppose that consumption or purchase can only be made in cash so that Cash-in-advance (CIA) constraint applies only on consumption good. The preference of the representative agent is

given by

$$U = \sum_{t=0}^{\infty} \beta^t lnc_t$$

The purchase of consumption good at time t is subject to the CIA constraint: ptct ≤ mt + wt.

Where Pt is the price of consumption good, Qt is the price of bonds, mt is nominal money balance that the household carried from the previous period and wt lump-sum-transfer equal to at the beginning of period t.

The budget constraint for the household in any period t is:

$$c_{t+} \frac{m_{t+1}}{P_t} + \frac{Q_t B_{t+1}}{P_t} \leq \frac{m_t}{P_t} + w_t + \frac{B_t}{P_t}$$

where Bt+1 is the total units of nominal bond demanded at time t.

Find the representative agents' optimization problem for (ct; mt+1; Bt+1) in order to maximize his inter-temporal utility function above subject to the CIA constraint and the wealth constraint.

Show that Qt is constant that is expressed in ƛ1t and ƛ2t

Solution

The representative agent's problem is to choose the sequence of (ct; mt+1; Bt+1) in order to maximize his inter-temporal utility subject to CIA constraint , and the wealth/ budget constraint. Students should specify the problem as below and determine the First Order Conditions (FOCs)

$$\max_{c_t, m_{t+1}, B_{t+1}} \sum_{t=0}^{\infty} \beta^t lnc_t$$

Subject to

$$\sum_{t=0}^{\infty} \beta^t [\text{ƛc} \left(\frac{m_t + w_t}{P_t} - c_t\right) + \text{ƛm} \left(\frac{m_t + w_t}{P_t} + \frac{B_t}{P_t} - c_t - \frac{m_{t+1}}{P_t} - \frac{Q_t B_{t+1}}{P_t}\right)]$$

This is then specified as:

$$\max_{c_t, m_{t+1}, B_{t+1}} \sum_{t=0}^{\infty} \beta^t [lnc_t + \lambda_{ct}\left(\frac{m_t + w_t}{P_t} - c_t\right) + \lambda_{mt}\left(\frac{m_t+w_t}{P_t} + \frac{B_t}{P_t} - c_t - \frac{m_{t+1}}{P_t} - \frac{Q_t B_{t+1}}{P_t}\right)]$$

F.O.C

$$c_t: \frac{1}{c_t} = \lambda_{ct} + \lambda_{mt} \quad (1)$$

$$m_{t+1}: \frac{\lambda_{mt}}{P_t} = \frac{\beta}{P_{t+1}}[\lambda_{ct+1} + \lambda_{mt+1}] \quad (2)$$

$$B_{t+1}: \beta\ \frac{\lambda_{mt+1}}{P_{t+1}} = Q_t \frac{\lambda_{mt}}{P_t} \quad (3)$$

Combining (1) and (2) we get:

$$\lambda_{mt} = \beta\ \frac{P_t}{P_{t+1}c_{t+1}} \quad (4)$$

From (3)

$$Q_t = \beta\ \frac{P_t}{P_{t+1}} \frac{\lambda_{mt+1}}{\lambda_{mt}} \quad (5)$$

Combining (5), (4) and (2), and rewriting (1) as $c_{t+1} = \frac{1}{\lambda_{mt+1} + \lambda_{ct+1}}$

$$Q_t = \frac{\lambda_{mt+1}}{\lambda_{mt+1} + \lambda_{ct+1}}$$

1.5 Monetary Search Models

Kiyotak, N. and R. Wright (1993), A Search-Theoretic Approach to Monetary Economics, *American Economic Review*, 83, no. 1 (march): 63-77.

According to classical economists, the essential function of money is its role as a medium of exchange. The use of monetary exchange helps to overcome the difficulty associated with pure barter in economies where trade is not centralized through some perfect and frictionless market. The search theoretic equilibrium model of the exchange captures the "double coincidence of wants problem" with pure barter in a simple and natural way. This gives rise to a medium-of exchange role for fiat currency.

The Basic Search-Theoretic Model

The economy is populated by a large number of infinite-lived agents, with total population normalized to unity. There is also a large number of consumption goods. These consumption goods are indivisible and come in units of size one. These are real commodities, which are different from fiat money, which is an object that no one ever consumes and can be thought of as a collection of pieces of paper or certain types of seashells, for example, with no intrinsic value. A crucial feature of the model is that there is an exogenous parameter x, with $0 < x < 1$, that captures the extent to which real commodities and tastes are differentiated.

In particular, x equals the proportion of commodities that can be consumed by any given agent, and x also equals the proportion of agents that can consume any given commodity. If a commodity is one of those that can be consumed by an agent, then we say that it is one of his consumption goods. Consuming one of his consumption goods yields utility $U > 0$, while consuming other commodities (or money) yields zero utility.

Initially, a fraction M of the agents are each endowed with money while 1- M are each endowed with one real commodity, where $0 < M < 1$. Money may or may not have value. If it does, then it is convenient to assume that agents who are initially endowed with money are endowed with exactly one unit of real balances, so that in order to buy a real commodity they must spend all of their cash. There are two ways to guarantee that this is the case. First, and most straightforwardly, we can simply assume that the monetary object is indivisible, like the real commodities in the model. Then if money trades at all it must trade one-for one against a real commodity, and each agent with one indivisible unit of money will have one unit of real balances. Alternatively, we can assume that money is divisible, determine the price level endogenously for a given stock of nominal currency, and endow some agents at the initial date with exactly enough nominal currency to constitute a single unit of real balances. We begin with the former approach of assuming that money is indivisible and take up the latter, slightly more complicated, approach later. Money and commodities are costlessly storable. Money cannot be produced by any private agent, while real commodities can be produced according to the following technology. One unit of output requires two inputs: a consumption good and a random amount of time. That is, once an agent consumes, he enters a production process that yields one unit of one real commodity, drawn randomly from the set of all commodities, according to a continuous-time Poisson process with arrival rate $\alpha > 0$. Thus, a measures productivity in the sense of average output per unit time. Note that agents who have not consumed cannot produce. Furthermore, as is standard in the equilibrium search literature, we assume that agents cannot consume their own output. This assumption helps to simplify the presentation and to facilitate comparison with earlier models.

An agent who has just produced enters an exchange sector where he looks for other agents with whom to trade. Traders in the exchange sector meet pairwise and at random according to a Poisson process with constant arrival rate $\beta > 0$. When two traders meet, exchange takes place if and only if it is mutually agreeable, that is, if and only if both agents are at least as well off after the trade. Because there is a large number of anonymous agents, all trade is quid pro quo (there can be no IOU's or other forms of private credit). We also assume that there is a transaction cost ε in terms of disutility, where $0 < \varepsilon < U$, that must be paid by the receiver whenever any real commodity is accepted in trade. This transaction cost implies that a trader who is indifferent between holding two real commodities will never trade one for the other.

For simplicity, we assume that the transaction cost of accepting fiat money is zero. Since exchange takes place if and only if mutually agreeable, an agent with either one unit of real balances or one real commodity cannot acquire additional money or another commodity except by giving up his entire inventory. Furthermore, no agent in the exchange sector can produce anything until he trades for one of his consumption goods and consumes, given the specified technology. These observations have the following implication: if each trader starts at the initial date with either one unit of real balances or one real commodity, then in equilibrium all traders will always have either one unit of real balances or one real commodity. Agents with real commodities are referred to as commodity traders, while agents with fiat money are referred to as money traders. Let μ denote the fraction of traders who are money traders, so that a trader located at random has money with probability, μ and a real commodity with a probability $1 - \mu$. Individuals choose strategies for deciding when to accept various commodities and fiat money in order to maximize their expected discounted utility from consumption net of transaction costs, taking as given the strategies of others. We look for Nash equilibria.

We restrict attention to symmetric equilibria, where all agents and all real commodities are treated the same, and to steady-state equilibria, where strategies and all aggregate variables are constant over time. To construct the set of such equilibria, we describe some basic properties that they must satisfy, use these properties to describe an individual trader's best response correspondence, and determine its fixed points.

The first thing to note is that an agent always accepts a real commodity if it is one of his consumption goods, where upon he immediately consumes it and enters the production process. Also, we claim that a commodity trader will never accept a commodity that is not one of his consumption goods. This is due to the fact that in a symmetric equilibrium no real commodities are treated as special, and therefore, the probability of a trade offer being accepted by the next agent one meets is independent of the type of commodity one has. Hence, there is no advantage to trading one real commodity for another, and since there is a transaction cost, ε, unless a commodity is going to be consumed it will never be accepted.

This means that x is the probability that a commodity trader located at random is willing to accept any given commodity, and therefore x^2 is the probability that two commodity traders consummate a barter transaction. This is the "double coincidence of wants problem" with direct barter: not only do you have to meet someone with something that you want, this someone also has to want what you have.

The next thing to determine is whether individuals accept money. Let Π denote the probability that a random commodity trader accepts money and let π be the best response of a representative individual. We will solve the best-response problem using dynamic programming. Let V_j denote the payoff or value function for the individual in state j, where j =

0, 1, or m indicates that he is a producer, a commodity trader, or a money trader, respectively. Then, if r > 0 is the rate of time preference, equations are given by

$$rV_0 = \alpha(V_1 - V_0) \ldots\ldots\ldots\ldots \text{(1)}$$

$$rV_1 = \beta(1-\mu)x^2(U - \varepsilon + V_0 - V_1) + \beta\mu x max\pi(V_m - V_1) \ldots\ldots\ldots \text{(2)}$$

$$rV_m = \beta(1-\mu)\Pi x(U - \varepsilon + V_0 - V_m) \ldots\ldots\ldots \text{(3)}$$

Where: ε = small transactions cost

Equation 1: The flow return to a producer, rVo, equals the rate at which output is produced, α, times the gain from switching from production to exchange, V_1 - V_O.

Equation 2: The flow return to a commodity trader equals the sum of two terms. The first term is the rate at which he meets other commodity traders, β(1-μ), times the probability that both want to trade, x^2, times the gain from trading, consuming, and switching back to production, U- **ε** + V_0-V_1 . The second term is the rate at which he meets money traders, βμ, times the probability that a money trader wants to trade, x, times the gain from accepting money with probability π, where π is chosen optimally.

Equations 3: The flow return to a money trader equals the rate at which he meets commodity traders, β(1- μ), times the probability that both want to trade, Πx, times the gain from trading, consuming, and switching to production, (U- ε + V_O-V_m). The above dynamic program depends not only on the strategies of others, as represented by Π, but also on μ, the proportion of traders holding money. However, μ can be determined as a function of Π and the initial endowment of money, M. Let N_o, N_1, and N_m denote the proportions of the population who are producers, commodity traders, and money traders respectively. Then the model has a dynamic structure.

To determine its steady state, we equate the flow out of and into production:

$$\alpha N_0 = \beta(1-\mu)x^2 N_1 + \beta(1-\mu)\Pi x N_m \ldots\ldots\ldots \text{(4)}$$

If we use the fact that the N_j's sum to 1 and the fact that N_m = M (the number of money traders equals the number of agents endowed with money), (4) can be reduced to

$$M = \alpha\mu/(\alpha + \varphi) \ldots\ldots\ldots \text{(5)}$$

Where:

$\varphi = \varphi(\mu, \Pi)$ is defined by[1]:

$\varphi = \beta(1-\mu)[\mu x\Pi + (1-\mu)x^2]$ ……………………………….. (6)

Equation (5) is a quadratic in μ, and for any $M \in [0,1]$ and $\Pi \in [0,1]$ there will exist a unique value of, $\mu = \mu(M, \Pi)$ in [0,1] satisfying this equation. Furthermore, one can show that, $\mu(0,\Pi) = 0$, $\mu(1,\Pi) = 1$, $\partial\mu/\partial M > 0$ and $\partial\mu/\partial\Pi > 0$. Given $\mu = \mu(M, \Pi)$, the unique steady state is fully described by:

$N_0 = \varphi/(\alpha + \varphi)$ and $N_1 = (1-\mu)\alpha/(\alpha + \varphi)$ …………………(7)

However, for the purpose of analyzing the above dynamic program, μ, summarizes all the agent needs to know about the steady state. Substituting $\mu = \mu(M, \Pi)$ into (1)-(3), then, given M, this dynamic program defines a correspondence from Π to best responses, μ. The set of equilibria is the set of fixed points of this correspondence.

To characterize this set, if Π < x then (1-(3) imply that $V_m < V_l$, which implies that the best response is $\pi = 0$. Intuitively, if money is being accepted with a lower probability than a barter offer, then it is harder to trade using money than barter, and so the best response is never to exchange a real commodity for money. Second, if Π > x, then (l)-(3) imply that $V_m > V_1$, which implies π = 1. If money is being accepted with a greater probability than a barter offer, then it is easier to trade using money, and so the best response is to exchange a real commodity for money whenever possible. Finally, if Π = x, then (l)-(3) imply that $V_m = V_1$, which implies that π can be anything in [0, 1]. If monetary exchange and barter are equally easy then traders are indifferent between having money and real commodities, and they could accept money with any probability. Based on these results, there are exactly three equilibria: Π = 0, Π = 1, and Π = x.

The equilibrium with Π = 0 will be called the nonmonetary equilibrium. In this case, agents expect that money will be valueless, so they never accept it, and this expectation is self-fulfilling. The equilibrium with Π = 1 will be called the pure-monetary equilibrium. In this case, agents expect that money will be universally acceptable, and so they always take it, and this expectation is self-fulfilling. F inally, the equilibrium with Π = x will be called the mixed-monetary equilibrium. In this case, agents are indifferent between accepting and rejecting money as long as other agents take it with probability Π = x, and so partial acceptability can

[1] Note φ can be interpreted as consumption per trader per unit time: it is the rate at which a representative trader meets commodity traders, $\beta(1-\mu)$, times the probability that a deal is reached, which is the probability that the representative trader has money and they trade, $\mu x\Pi$, plus the probability that the representative trader has a real commodity and they trade, $(1-\mu)x^2$.

also be self-fulfilling. Alternatively, a symmetric mixed-strategy equilibrium where all agents accept money with probability x could be reinterpreted as a non-symmetric pure strategy equilibrium, where a fraction x of agents accept money with probability 1 while the rest accept it with probability 0.

1.6 Asset Price Uncertainty and the Speculative Demand for Money

(a) Keynesian Theory of Demand for Money

Keynes's development of Cash Balance approach to the problem of the demand for money now forms the basis of the treatment of the subject in macroeconomics. Keynes distinguished 3 main motives for holding money, namely, the transactions, precautionary and speculative motives. The transactions and precautionary motives are derived from money's use in facilitating exchanges, while the speculative motive is derived from money's use as an asset, as a store of value.

Keynes treatment of the transactions and precautionary motives adds nothing particularly new to the analysis developed by the Cambridge school, but rather develops that analysis a little further. The transaction demand is simply the demand for money that is necessary to carryout normal current transactions, i.e. those which are routine and predictable. These transactions are carried out both by private persons and businesses so that Keynes divided his transactions motive into an income motive and a business motive. The income motive is that transactions motive applied to private persons, a motive arising out the absence of perfect synchronization of personal payments and receipts. The strength of this motive depends according to Keynes, largely on the size of incomes and the length of time between the receipt of income and its being paid out. The business motive refers to the desire on the part of businesses to hold cash in order to bridge the interval between the incurring of costs and receipt of the proceeds from sales. The strength of this motive depends on the value of current output and hence on current income and on the numbers of hands through which output passes.

Keynes analysis of the precautionary motive is a straightforward restatement of the Cambridge security motive. He suggested that people also find it prudent to hold some cash in case they are not able to realize other assets quickly enough to be of used to them for those classes of payments that cannot be considered regular and planned, such as paying unexpected bills, making purchases at unexpectedly favorable prices, and meeting sudden emergencies caused perhaps by accidents or health. This he called the precautionary motive for holding money and suggested that the demand for money arising from it also depends, by and large, on the level of income.

These 2 motives were also regarded by Keynes as being influenced by 2 other factors. First, there is the cost of and the ability to obtain money at the point in time when it is required to spend; if a person is certain that he can borrow money cheaply at the point in time in the future when he knows he will be incurring expenditure, he may prefer to do this rather than carry a cash balance for the intervening time period. Secondly, there is the opportunity cost of holding money.

Speculative demand for money

This is demand for money in respect to future level of the rate of interest. People want to hold money, Keynes said not only for transacting current business but also as a store of value or wealth because of the existence of uncertainty as to the future of the rate of interest. Once the future rate of interest is uncertain people have the opportunity to speculate in the hope of securing profit from knowing better than the market what the future will bring forth. By uncertainty Keynes did not appear to mean uncertainty on the part of any individuals as to the future course of the rate of interest. Each individual is seen as being quite clear in his or her own mind as to what is going to happen to the rate of interest, but individual views differ from person to person. If there were no differences of opinion, then nobody would be able to profit from knowing better than the market what the future will bring forth: nobody's view would differ from the market view. In this analysis of the speculative motive, Keynes considered only one alternative to money as a store of value, namely bonds.

A bond is an asset that carries with it the promise to pay its owner a certain income per annum, fixed in money terms, and the decision to buy a bond is a decision to buy a claim to such a future stream of income. How much any individual agent is willing to pay for a bond, hence the market value of the bond, depends critically on the rate of interest because prospective purchasers will wish to earn at least the going rate of interest on that portion of wealth being held in the form of bonds. For instance, suppose the government can issue a bond of infinite term to maturity that promises the owner a fixed period payment of 1$ from now to infinity. If the rate of interest is r, how much would an investor be willing to pay for such a bond? Obviously, the price of the bond, P_B would be exactly equal to the present value of the stream of income derived from the bond, or, in continuous time: $P_B = \int_0^{\infty} 1e^{-rt}\ dt \ = (r^{-1}e^{-rt}) = r^{-1.}$

If there are Y of such bonds, $P_B = Y/r$. Therefore, if the rate of interest is 5%, individual will be willing to pay up to, but not more than 100 dollars for a bond that offers an income of 5 dollars per annum in, perpetuity. If the rate of interest is 10%, no one will be willing to pay more than 50 dollars for the same bond. It follows, then, from the very nature of bonds that changes in the rate of interest involve changes in their price.

A bond-holder expecting the rate of interest of rise, anticipates a capital loss on the bond which must be set against the interest income from the bond. In some cases, the capital loss will more than wipe out the interest income, in others will be less than the interest income, while in other cases it will exactly equal the interest income. In the first case bond-holding would offer a negative income, in the second a positive income, and in the third a zero income. Considering the choice between holding money and bonds, it is clear that in addition to offering the attraction of an interest income, which money does not offer, bonds when the rate of expected to fall, also offer to their owners the possibility of making capital gains. In such circumstances, they are particularly attractive to hold but when the rate of interest is expected to rise, the situation is the opposite. In the case where bond- holding offers a zero income the person will be indifferent as between bonds and money as stores of wealth. It is what people think is going to happen to the rate of interest which will determine whether they store their wealth in bonds or money.

What a person thinks is going to happen to the rate of interest will depend upon the relationship of the current rate of interest to the rate that the person thinks is the normal one. Every person is thought to have in mind an idea as to what is the safe or normal level for the rate of interest. This normal rate will vary from person to person.

Price of bond = $P_b = \frac{Y}{r}$

Where Y is the yield usually stated as a percentage of face value of the bond. The expected percentage capital gain (g) is the percentage increase in price from the purchase price P_b to the expected sale price P_b^e .

$$g = \frac{P_b^e - P_b}{P_b}$$

But $P_b^e = Y/r^e$ and $P_b = Y/r$

$$g = \frac{\frac{Y}{r^e} - \frac{Y}{r}}{\frac{Y}{r}} = \frac{r}{r^e} - 1$$

If $r > r^e$, the speculator anticipates a positive capital gain, while if $r < r^e$ he anticipates a capital loss. The total rate of return on bond (E) will be the sum of the market rate of interest at the time of purchase and the capital gains term. E = r + g implies that E = E = $r + \frac{r}{r^e} - 1$. If expected E > 0 the asset holder can be expected to put his liquid wealth into bonds. If expected E < 0 he will put his liquid wealth into money.

In the regressive expectations model, each person is assumed to have an expected interest rate r^e corresponding to some normal long-run average rate. If rate rise above this long-run expected rate, one expects them to fall and vice versa. Therefore, one's expectations are regressive.

Critical interest rate, which would give, net zero return i.e. E = 0:

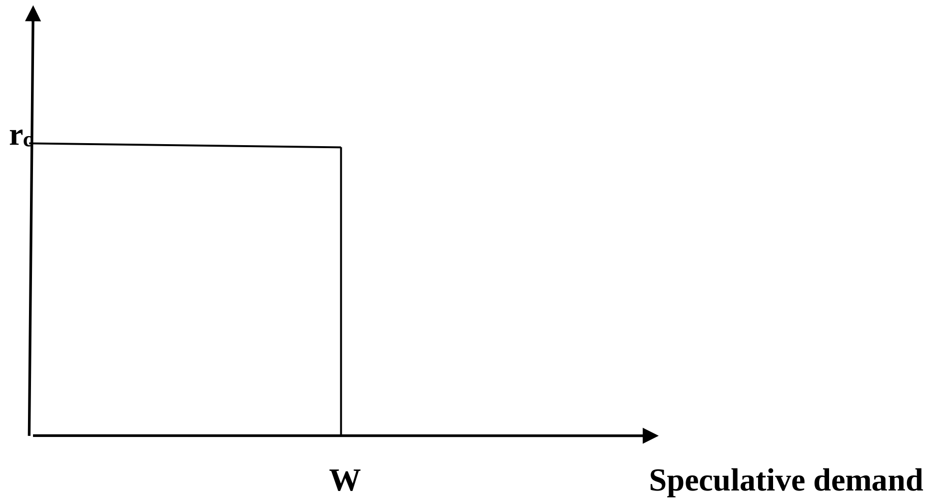
$$r + r/r^e - 1 = 0$$

$$r_c = r^e/(1 + r^e)$$

Individual's Demand for Money

This analysis implies that speculators will put the whole of their speculative balance either into money or into bonds. The choice between money is an all-or-nothing one. At current rates above the critical one, bond-holding will offer a positive return and so all speculative balances go into money. Diagrammatically, this means that the individual's speculative demand for money curve is a discontinuous step function.

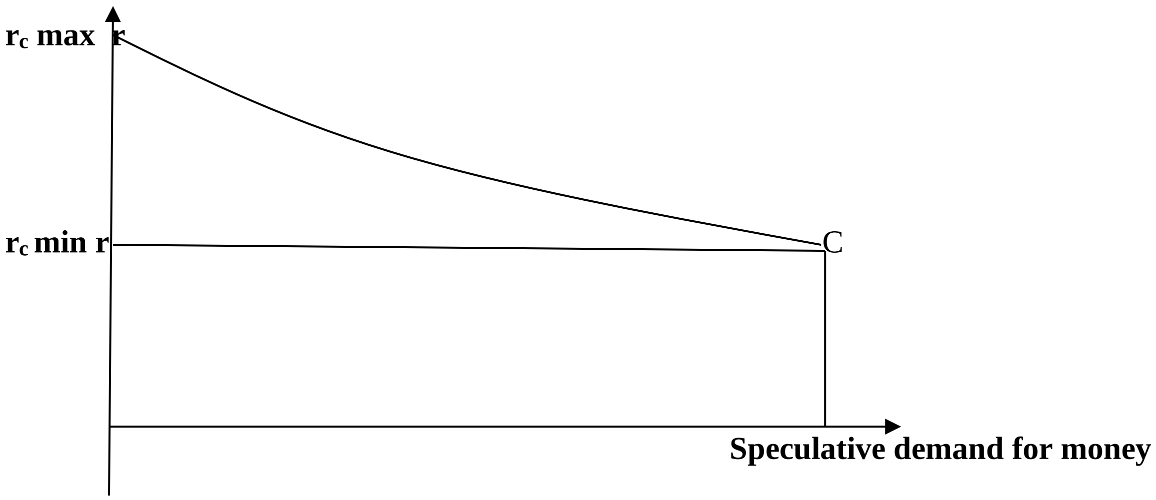
Interest rate



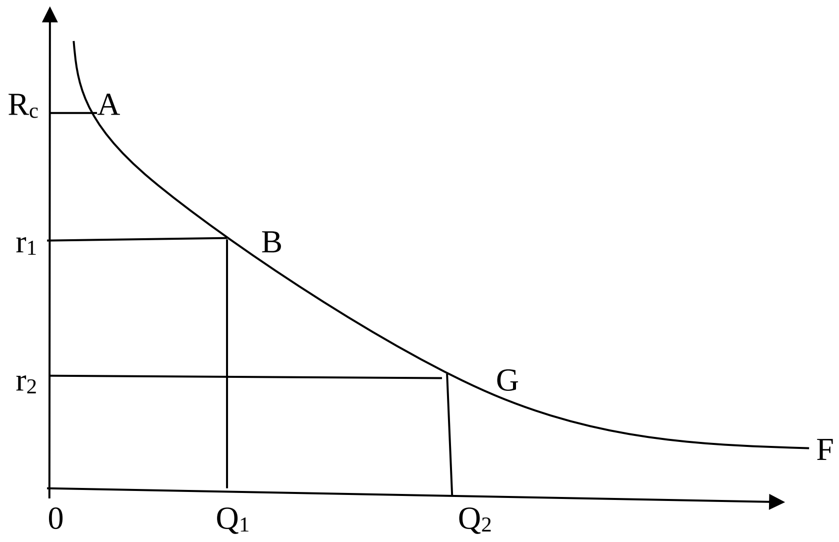
Where r is greater than r_c , the asset holder puts all of individual's total resources W into bonds, so that demand for money is zero. As r drops below r_c so that $E < 0$ and expected capital losses on bond outweigh the interest yield, the asset holder moves his entire liquid wealth into money.

Aggregate demand for money

Different individuals will, however, have different views as to what is the normal rate of interest, and consequently will have different critical values of the current rate. At any level of the current rate, all those people whose critical rate is below it will put their speculative balances into bonds, while all those whose critical rate is below it will put their speculative balances into money. With any given distribution of critical rates, the higher is the current rate, the greater will be the number of people who have critical rates below it and the smaller will be the amount of money going into speculative balances. Similarly, with any given distribution of critical rates there must be a maximum critical rate, the highest held by anybody. At current rates there must be a maximum critical rate, everybody will put the whole of their speculative balances into bonds and no money will be held at all. Equally, there will be a minimum critical value, the lowest one held by anybody, so that at current rates below it, everyone holds all of their balances in money. At current rates above max r_c all speculative balances go into bonds, below min r_c all speculative balances go into money. In particular, the lower the current rate of interest rate, the more people there are who will want to hold all of their speculative balances in the form of money, believing that the interest is going to rise sufficiently rapidly to cause a capital loss that will wipe out the interest income from bond-holding. At aggregate level, there will therefore be some minimum critical level at which everybody desires to hold cash and the demand for cash at the rate of interest is perfectly elastic in the sense that whatever the amount of money available for the speculative motive, it will be demanded and disappear into idle speculative hoards. The shape of the curve between B and C will depend upon the numbers of people associated with each critical rate, and the distribution of speculative balances between those individuals.



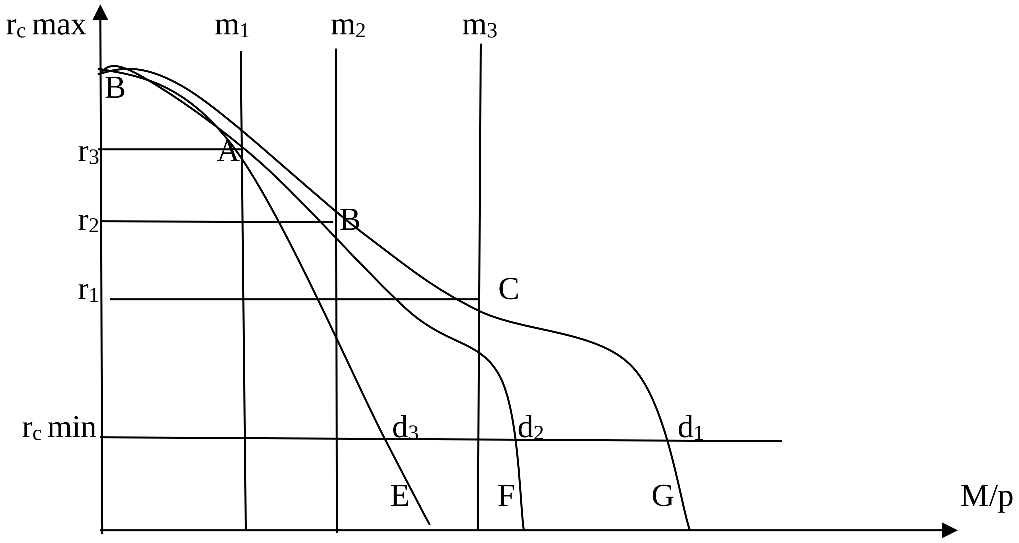
So far the assumption has been that individuals have fixed levels of speculative balances, but if wealth consist partly bonds, then a change in r by changing the market value of those bonds, alters the total value of wealth that could be held in speculative balances. An increase in the rate of interest, by lowering capital values of bonds, reduces the value of wealth available for holding as speculative balances.



Suppose the current rate of interest is above r_c; then the individual is holding all of his speculative balance in bonds. If the current rate of interest now falls to say r_1 then the individual would move out of bonds into cash. At the current rate r_1, the market value of the person's bond-holding is greater than it was when the rate was above r_c. The encashment of his bonds at the rate r_1 enables him to hold $0Q_1$ money. If the current rate had fallen to r_2 rather than r_1, then the individual would be able to encash his bonds for $0Q_2$. Thus, the individual's speculative demand for money would appear to be ABGF. This explanation needs qualifying, however, suppose that the current rate of interest is marginally above the individual's critical rate, so that this speculative balance is held wholly in bonds. If the current rate now falls to a level marginally below the person's critical rate, he will move all of his speculative balance into cash. Once all his speculative demand curve becomes, in other words, perfectly inelastic at the rate of interest at which the individual's speculative balance is held wholly in cash. If for example, the current rate were r_1, with the individual holding all of his balance in money, then a fall in the current rate to r_2 would not increase the individual's speculative demand for money to $0Q_2$.

The aggregate demand curve for speculative balances which allows for wealth effects will be flatter than one which does not. This is demonstrated by considering the consequences of increasing the money supply. Before that, however, we must recognize that Keynes introduced a theoretical limiting case into his analysis of the speculative motive, namely absolute liquidity preference (liquidity trap). As already noted, the lower the current rate of interest, the more people there are who will want to hold all of their speculative balances in the form of money, believing that the interest rate is going to rise sufficiently rapidly to cause a capital loss that will wipe out the interest income from bond holding. There will therefore be some low level of the current rate-the minimum critical rate of the community-at which everybody desires to hold cash; the demand for cash at this rate of interest is perfectly elastic in the sense that whatever the amount of money available for the speculative motive it will all be demanded and disappear into idle speculative hoards.

Curve BE, BF and BG are three aggregate speculative demand curves each drawn on the assumption of a given level of aggregate wealth. If the money supply is M_1, and the relevant demand curve is BE, then the equilibrium position is at A and the rate of interest is r_3. Suppose the money supply is increased to M_2. With an unchanged demand curve, the interest rate would fall to r_2. But the fall in the interest rate increases aggregate wealth and shifts the demand curve to the right, where a new equilibrium position is given by point B with interest rate r_2. Similarly, a further increase in the money supply to M_3 again depresses interest rates, thereby shifting the demand curve to BG and establishing a new equilibrium position at C, with interest rate r_3. If points of equilibrium like those of AB and C are joined up, a demand curve for speculative balances that allow for interest rate and wealth effects is derived. Such a curve will approach minimum r_c asymptotically becoming flatter and flatter as interest rate falls, and thus having the shape traditionally assigned to the demand for money curve.



The wealth effect of the demand for money

The basis of Keynes' speculative motive, then, is uncertainty. Uncertainty in the sense that different people will hold different views as to what is going to happen to the current rate of interest. Each person will believe that it is going to move towards what he regards as their expectations change, expectations being subject to a very large number of influences. In summary, according to Keynes, the amount of money for transactions and precautionary motives, (L_1) depends on the level of incomes; the amount held for speculative motive, (L_2) depends mainly on the rate of interest and expectations. Thus, $M = L_1(Y) + L_2(r)$. Keynes' analysis of the demand for money is clearly, therefore, divided into two parts, one part approaching money simply as a means of payment, the other as an asset. The significant innovation of Keynes' analysis was to show the demand for money was interest-elastic and that at some low rate of interest of interest it might become perfectly so. There have, however, been several criticisms of these views.

1. If the money market remained in equilibrium for a long enough period, people should begin to adjust their expected interest rates to correspond to the actual prevailing interest rate. They would all tend to adopt eventually the same critical interest rate as time passes so that the speculative motive would zero.

2. The model assumes that individuals hold either all bonds or all money, never a mix of the two. The negative slope of the aggregate demand curve is due to the fact people disagree about the value of expected interest rate and thus in their critical rates r^c. The reality is however that individuals hold mixture of assets – money and bonds.

(a) Tobin's Portfolio Balance Approach to Money Demand

Tobin attempts to show how people's desire to hold money may be derived from their attitude towards the risk involved in holding bonds. Uncertainty as to the future rate of interest means uncertainty as to the future capital value of bonds. People holding bonds are consequently unsure whether they will make a capital gain or capital loss. Individual has a saving balance which he can keep on in the form of money or invest in bonds. If he keeps it in the form of money, then on the assumption of a stable price level there is no risk attached. But while there is no attached to money holding there is no possibility of an income either. If the individual invests his savings balance in bonds, then he does have a money income in the form of the interest the bonds will earn him. There may also be income in the form of a capital gain should the rate of interest fall while he is holding the bonds. But if the interest rises, he will incur a capital loss. With bond holding, therefore, one has the attraction of a return in the form of a money income, but the risk that the income may turn out to be negative if the bond has to be sold for a capital loss which more than wipes out the interest income. The problem for the individual is in the balancing of risk and return elements in bond holding.

For most people the prospect of money income is desirable, while risk is undesirable. Suppose a person is offered the choice between a perfectly safe US $ 100m or the 50-50 chance of either US $ 50m or US $ 150m. Which will be chosen? If he chooses the risky option, he will have an equal chance of making 50m more or 50m less than the safe option. But if we assume diminishing marginal utility of money income, then extra utility derived from the last 50m of the 150m would be less than utility loss on the 50m by which the less desirable outcome of the risky option falls short of the safe choice. Most people confronted by this choice would probably choose the safe US $ 100m. But of course, not everybody would do so. Some people would be attracted by the possibility of the large gain and would prefer a choice involving this possibility to the more certain but lower income.

Assuming the individual has a savings portfolio which he wishes to divide between cash and bonds; as cash provides no return, the return from the total portfolio will depend on the fraction held in bonds. The return itself will contain two elements, the interest income and the capital gain which will be negative in the case of a capital loss. The expression for total return $E = r + g$. But rather than some fixed expected capital gain, the asset holder has a whole spectrum of expected capital gains, each with a probability of its occurrence attached. He is not certain of the value of g he expects but has an implicit distribution. These gains are distributed around some central value. If the probabilities of capital gains are distributed normally, then standard deviation, r_g, of the probability distribution of capital gains measures uncertainty of risk.

If the asset holder is putting B dollars of his assets into bonds, his expected total returns

$$R^e_T = B.E$$

$$R^e_T = B(r + g).$$

But g is uncertain because the individual is uncertain what is going to happen to the rate of interest. There will be a number of possible values of g but the mean value of these possibilities provided a useful single measure. Thus $R^e_T = B(r + g^-)$. If we make the assumption that the individual thinks there is an equal chance of a capital gain and loss of the same size so that expected $g = 0$, then $R^e_T = Br$.

As the measure of the risk involved in bond-holding, Tobin takes the standard deviation of the return on bond is r_g, which is a statistical measure of possible returns around the mean value $\bar{g}$. A high standard deviation means roughly that there is a high probability of large deviations, both positive and negative, from the mean, while a low standard deviation means a low probability of high deviation from the mean. Because no risk is attached to holding money, the risk surrounding the total return depends, of course, on the risk surrounding the capital gain/loss from bond-holding, and the proportion of the total portfolio held in risk-bearing bonds. Since all bonds are similar, then the total standard deviation of bond holdings is given by

$$\delta_T = B.r_g \text{ and therefore } B = \delta_T / r_g$$

The expected return and the risk are both dependent on the proportion of the total balance held in bonds. As this proportion increases, both the return and the risk attached to the total portfolio increase. In other words, in order to enjoy a greater expected return the portfolio holder must also assume more risk. Thus,

$$R_T^e = \frac{\delta_T}{r_g}(\mathrm{r} + \bar{g})$$

$$\frac{dR_T^e}{d\delta_T} = \frac{r + \bar{g}}{r_g}, \text{ and}$$

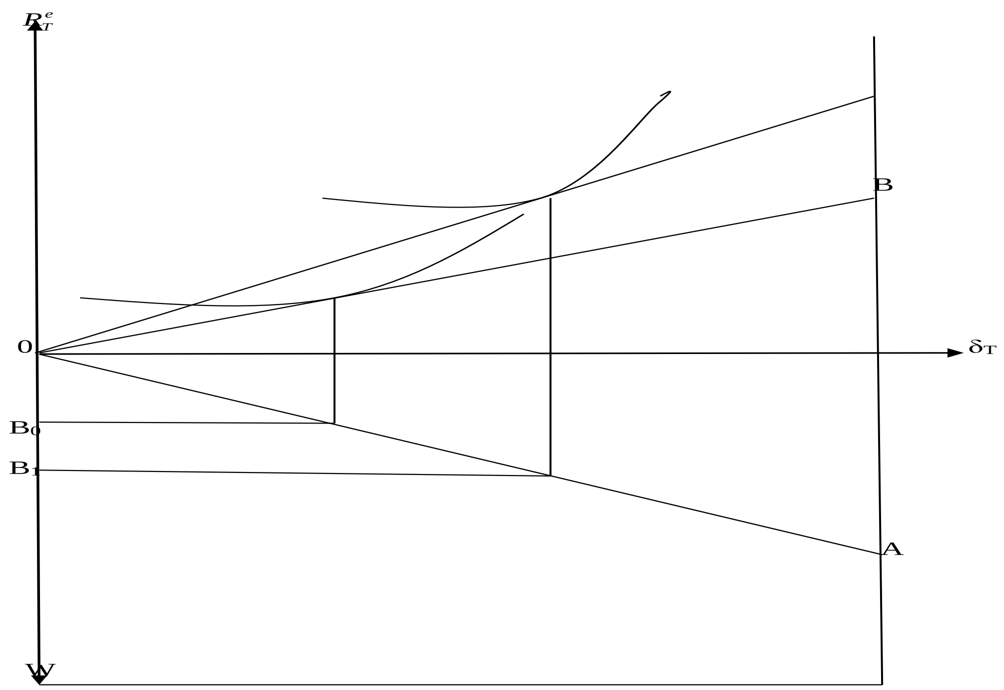
$$\frac{dB}{d\delta_T} = \frac{1}{r_g}$$

OB equals the opportunity curve, tracing the combinations of risk and expected return available to the investor. OA traces out the relationship between the proportion of the portfolio held in bonds and the risk attached to the portfolio. OW = total liquid wealth divided into money and bonds and Money holding = W- B

Each indifference curve joins up the various combinations of risk and return between which the portfolio- holder is indifferent. The slope of the indifference curves will depend upon the portfolio- holders' attitude toward risk. If the individual is a risk-averter, somebody who finds risk unattractive, then he will only assume more risk if at the same time it increases his expected return. Consequently, incremental increases in risk will require bigger and bigger increases in expected return to leave the individual indifferent between the various combinations, making the indifference curves convex downwards.

For any given value of δ_T we can locate the value of B and subtracting it from W (wealth) we obtain money holding. In order to locate the individuals equilibrium risk δ_T and expected rate of return R_T^e we confront the technical budget constraint with the individual's utility function showing trade-off between risk and return. The slope of the indifference curves will depend upon the portfolio-holder's attitude towards risk. If we assume that the individual is a risk-averter diversifier, somebody who finds risk unattractive, then he will only assume more risk if at the same time it increases his expected return. Consequently, incremental increases in risk will require bigger and bigger increases in expected return to leave the individual indifferent between the various combinations, making the indifference curves convex downwards. The diversifier will be at optimal where he reaches the highest indifference curve at point e_0 , i.e. the expected return and risk of his portfolio will be determined by the point of tangency of the budget line with the highest possible indifference curve.

AS interest rate increases from r_0 to r_1 to r_2 the slope of the budget line rotates upward. At any given level of risk, return will be increased as r rises. As r increases, the budget line touches successively higher indifference curves. This traces out the optimum portfolio curve connection the points of tangency. An increase in g will have the same effect as in increase in r and an increase in δ_g reduces bond holding.



The indifference curves of gamblers, the risk lovers, will however be negatively sloped. Such individuals are prepared to accept a lower expected return in order to have the chance of unusually high capital gains associated with a high risk. For instance, the risk-lover will not diversify his portfolio, but will hold his entire portfolio in bonds, being attracted by the possibility of large capital gains. His preferred combination will always be a corner one. The risk-averter plunger will also seek a corner maximum, but it may be all money or all bonds one.

1.7 Buffer-Stock Models of Money Demand

The theoretical analysis of buffer stock models extends the inventory analysis of the transactions demand for money to the case of uncertainty of net payments (payments less receipts). The buffer stock models allow short-run money balances to fluctuate within a band which has upper and lower limits, also known as thresholds, or fluctuate around a long-run desired money demand.

There are basically two versions of buffer stock models. In one of these, a “policy decision” is made a priori by the individual that cash balances will be allowed to vary within an upper (M_{max}) and a lower limit (M_{min}). When the autonomous – that is, independent of the decision to invest in bonds or disinvest from bonds – net receipts cause the accumulated cash balance to hit the upper limit M_{max}, action is taken to invest a certain amount in other assets, say bonds, thereby reducing cash holdings suddenly by the corresponding amount. Whenever the autonomous net payments deplete the cash reserves sufficiently to reach the minimum permitted level M_{min}, action is taken to rebuild them by selling some of the bonds. This lower limit can be zero or a positive amount, depending upon institutional practices such as minimum balances required by banks, etc.

Such buffer stock models with a pre-set band belong to the (Z, z) – with Z as the upper limit and z as the lower one – type of inventory models and are called “rule models”, where the rule specifies the adjustment made when the money balances hit either of the limits.

In such rule models, money balances can change because of positive or negative net payments or because of action taken by the agent to reduce them when they reach the upper limit or increase them when they reach the lower limit. The former can be designated as “autonomous” or “exogenous” changes and the latter as “induced” changes in money balances. In the former, the change occurs even though the agent’s objective is not to adjust his money holdings. In the latter, the agent’s intention is to adjust the money balances since they have moved outside the designated band.

The second type of buffer stock models is called “smoothing or objective models.” In these, the objective is to smooth movements in other variables such as consumption or expenditures and bond holdings. Unexpected increases in income receipts or decreases in payments would be added to money balances acting as the “residual” inventory or temporary abode of purchasing power until adjustments in expenditures and bond holdings can be made. Conversely, unexpected decreases in income receipts or increases in payments would be temporarily accommodated by running down money balances, rather than through an immediate cutback in expenditures or sales of bonds. The reason for thus treating money holdings as a residual repository of purchasing power is that the cost of small and continual adjustments in such balances is assumed to be lower than in either expenditures or payments, or in bond holdings, so that temporarily allowing such balances to change is the optimal strategy. In such smoothing models, actual balances fluctuate around their desired long-run demand, but there are no pre-set upper and lower limits as in the case of the rule models. The distinction between the autonomous and induced (causes of) changes in money balances applies in both smoothing and rule models.

BUFFER STOCK RULE MODELS

The Rule model of Akerlof and Milbourne (1980)

Model assumes a lump-sum receipt of Y at the beginning of the period and expenditures of C at a constant rate through the period. Model also assumes that $C \leq Y$, with saving $S = Y - C$. Saving during the period is added to money balances until the latter reach the set upper limit, at which time action is taken to decrease them to C through their partial investment in bonds, so that they are expected to be exhausted by the next period.

Designate the upper limit as Z and the lower one as z, with the latter taken to be zero for simplification. The agent wishes to start each period with the amount C, which will therefore be the desired amount at the beginning of each period. The actual amounts held at the beginning of the ith period equal C +nS, as long as $(C + nS) \leq Z$. If the upper limit is reached after n periods, we have:

$$C + nS \geq Z \geq C + (n-1)S \quad \text{…………………………………………… (1)}$$

So that:

$$n \geq \frac{Z-C}{S} \geq n-1 \quad \text{……………………………………………………….. (2)}$$

Hence, the maximum and minimum values of n are:

$$n_{max} = \left[\frac{Z-C}{S}\right] + 1 \quad \text{……………………………………………………. (3)}$$

$$n_{min} = [\frac{Z-c}{S}] \quad \text{…………………………………………………… (4)}$$

The amount C is spent evenly during the period so that the average amount of cash balances corresponding to it is C/2. When saving S is added to money balances in a period, this amount is held from the beginning of the period to its end, so that the average cash balances corresponding to it are S. Therefore, the sequence of money balances at the end of each period is:

$$\left\{\frac{C}{2}, \frac{C}{2} + S, \frac{C}{2} + 2S, \dots, \frac{C}{2} + (n-1)S\right\} \quad \text{………………………………………(5)}$$

Which equals:

$C/2\{1, 1 + S, 1 + 2S, \dots, 1 + (n-1)S\}$

Let n be the number of periods before an induced transfer takes place. Then, over n periods, the average balance between induced adjustments is:

$$M^d \equiv \frac{1}{n}[\frac{C}{2} + \left(\frac{C}{2} + S\right) + \left(\frac{C}{2} + 2S\right) + \cdots + \left\{\frac{C}{2} + (n-1)S\right\}] = \frac{C}{2} + \frac{S(n-1)}{2} \quad \ldots\ldots\ldots (6)$$

Using (3) and (4) to eliminate n in (6) implies the minimum and maximum values of money balances as:

$$M_{max} = Z/2 \quad \ldots\ldots (7)$$

$$M_{min} = (Z - S)/2 \quad \ldots\ldots (8)$$

Hence, the average of the money balances held as a buffer stock, designated as M^b, is:

$$M^b = \frac{1}{2}(M_{max} + M_{min}) = Z/2 - S/4 \quad \ldots\ldots (9)$$

So that:

$$\frac{\partial M^b}{\partial Y} = -(1/4)(\frac{\partial S}{\partial Y}) < 0 \quad \ldots\ldots (10)$$

Where $\partial S/\partial Y$ is the marginal propensity to save, which is positive. Hence, $\partial M^b/\partial Y$ is negative. This is a surprising result. Its intuitive explanation is that, as income rises, the upper threshold is reached more quickly, so that the interval before the money holdings are run down by an induced adjustment is shortened. As a result, the richer agents review their money and bond holdings more often than those with lower incomes, ceteris paribus, and will hold less balances on average.

However, since the limits z and Z were assumed to be exogenously specified in the above model, the impact of increases in Y on them is not incorporated in (9). Transactions demand analysis implies that both of these limits would be positive functions of the level of expenditures. Therefore, the impact of a rise in income would have a positive and a negative component, with the net impact being of indeterminate sign unless a fuller model were specified. Another limitation of the above model is that it does not distinguish between the expected increases in income and the unexpected ones. The (z, Z) concept is more appropriate to the latter than to the former.

Akerlof and Milbourne extended their preceding model to the case of the uncertainty of net payments by assuming that the agent buys and pays for durable goods at uncertain times, with p as the probability of making such a purchase. For this case, under their simplifying assumptions that include S = sY , where s is the constant average propensity to save, was that:

$$M^b = Z/2 - s(1+p)(Y/4) < 0 \quad \ldots\ldots (11)$$

where p is the probability of payments (for a durable goods purchase). Eqn 11 implies that:

$$\partial M^b = -(S/4)[1 + p + Yp'(Y)] \quad \ldots\ldots (12)$$

where p' = ∂p/∂Y. Assuming p' to be positive – that is, the probability of buying durable goods increases with income – (12) implies that the income elasticity of money balances is again negative.

The model was meant to encompass both the transactions and precautionary demands. The agent's desire to finance an amount of transactions C out of income receipts Y creates his transactions demand, while the uncertainty of payments adds an additional precautionary demand. However, this framework does not properly capture the transactions demand since it ignores the dependence of (z, Z) on total expenditures and does not make a distinction between expected and unexpected changes in income. Its implication of a negative income elasticity of money demand must therefore be accepted as reflecting the influence of saving, and especially unexpected saving, on money demand, with consumption – and hence permanent income – being held constant.

The Rule Model of Miller and Orr (1966)

Miller and Orr (1966) assumed that net receipts – which would be net payments for a negative value – of x at any moment follow a random walk with a zero mean over each period (e.g., a "day"). Assume that in any time interval (e.g. an "hour") equalling 1/t(t = 24), x is generated as a sequence of independent Bernoulli trials. The individual believes with a subjective probability of p that he will have net receipts of x during each time interval (hour) or net payments of x with a probability of (1−p), so that, over an hour, the probability of an increase in money holdings by x is p and that of a decrease by x is (1−p).

Cash holdings over a decision period of T periods will have a mean and standard deviation given by:

$$\mu_T = Ttx(p-q) \quad \text{……………………………………………………………… (13)}$$

$$\sigma^2{}_t = 4Ttpx^2 \quad \text{……………………………………………………………… (14)}$$

where:
x = net receipts per hour
μ_T = mean cash holdings over T periods
σ_T = standard deviation of cash holdings over T periods
p = probability of positive net payments
q = probability of negative net payments (= 1−p)
t = number of sub-intervals (hours) in each period (day)
T = number of time periods up to the planning horizon.

For simplification, p was assumed to be 1⁄2, so that:

$$\mu_{T=0} \quad \text{……………………………………………………………………… (15)}$$

$$\sigma^2{}_T = Ttx^2 \quad \text{………………………………………………………… …..(16)}$$

Since the variance of changes in cash holdings ($\sigma^2{}_T$) during T periods is Ttx^2, the variance (σ^2) of daily changes in balances (over the day) is:

$$\sigma^2 = \sigma^2{}_T/T = tx^2 \quad \text{……………………………………………. (17)}$$

The cost of holding and varying cash has two components: the interest cost of holding cash rather than bonds, and the brokerage cost of making deliberate changes in cash holdings. The per period (daily) expected cost is:

$$E(TC) = B_0E(N)/T + RM \quad \text{……… (18)}$$

Where:
$E(TC)$ = expected cost per period of holding and managing cash
M = average daily cash balance
$E(N)$ = expected number of transactions between money and bonds over T periods
B_0 = brokerage cost per transaction
R = interest rate per period (day) on bonds

The firm is taken to minimize (49) with respect to the upper limit Z and the lower limit z on cash balances. Under certain specific assumptions:
$E(N)/T \rightarrow 1/D(z,Z)$

where D is the mean of the time intervals separating portfolio transfers between bonds and cash, and that:

$$D(z,Z) = z\,(Z - z)/tx^2 \quad \text{……….. (19)}$$

Further, the steady–state distribution of money balances during the day has a discrete triangular distribution for $p = 1/2$, so that the average balances M are given by:

$$M = (Z + z)/3 \quad \text{………(20)}$$

Hence, using the maximum value of $E(N)/T$, (20) can be restated as:

$$E(TC) = B_0\,tx^2/zw + R(w+2z)/3 \quad \text{……… (21)}$$

Where $w = Z - z$. w is thus the width of the band. Setting the partial derivatives of (21) with respect to z (with w constant), and w (with z constant) equal to zero, yields:

$$\partial E(TC)/\partial z = B_0\,tx^2/z^2w + 2r/3 = 0 \quad \text{……… (22)}$$
$$\partial E(TC)/\partial w = -B_0\,tx^2/zw^2 + R/3 = 0 \quad \text{……… (23)}$$

Which yield the optimal values z^* and w^* as:

$$z^* = (3B_0\,tx^2/4R)^{1/3} \quad \text{………(24)}$$
$$w^* = 2z^* \quad \text{………(25)}$$

Since $w = Z - z$, (25) implies that the optimal upper limit Z^* will be:

$$Z^* = 3z^* \quad \text{……… (26)}$$

Which is independent of the interest rate and the brokerage cost, though, by (24) to (26), the *absolute* width of the band does depend upon these variables.

From (26), the mean buffer stock balances M^b under the assumptions of this model are given by:

$$M^b = (Z + z)/3 \quad \text{……… (27)}$$

Therefore, the average optimal buffer stock balances M^{b*} derived from (24), (26) and (27) are:

$$M^{b*} = \frac{4}{3}\left(\frac{3B_0 t x^2}{4R}\right)^{1/3} = \frac{4}{3}\left(\frac{3B_0 \sigma^2}{4R}\right)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

since $\sigma^2 = tx^2$.

In (28), the average demand for money depends upon the interest rate and the brokerage cost and upon the variance of net payments. The elasticity of the average demand for money with respect to the variance of income is 1/3, and with respect to interest rates it is (−1/3).

However, since there does not exist a precise relationship between $\sigma 2$ and Y, there will not exist a precise income elasticity of Mb with respect to Y. To illustrate, if we are dealing with a firm's demand for money and Y is its income from sales, a proportionate increase in this sales income due to a proportionate increase in all receipts and payments by it, with their frequency unchanged, will increase x proportionately, so that $\sigma^2 = tY^2$, implying from (28) an income elasticity of 2/3. But if the amount of each receipt and payment does not change but their frequency is increased, so that t increases proportionately with Y such that $t = \alpha Y$, we have $\sigma^2 = \alpha Y x^2$, thereby implying from (28) an income elasticity of 1/3. The implied range for the income elasticity of the average buffer stock balances becomes even larger than from 1/3 to 2/3 if the amounts of the transactions increase while their frequency decreases. The model extends the analysis of the precautionary demand for money to the case where there are fluctuations within upper and lower limits, with these limits derived in an optimizing framework. The existence of a range for the income elasticity of the average buffer stock balances, rather than a single value as for Baumol's transactions balances, is another empirically appealing feature of the model. These authors considered their model to be especially appealing in explaining the firms' demand for money.

BUFFER STOCK SMOOTHING OR OBJECTIVE MODELS

The smoothing model of Cuthbertson and Taylor

The basic partial adjustment model (PAM), in the context of a single period, often assumes that the adjustment in money balances involves two kinds of costs. One of these is the cost of deviations of actual balances from their desired amount. The second element is the cost of changing the current level of balances from their amount in the preceding period. The one-period first-order PAM assumes that the cost function is quadratic in its two elements,
as in:

$$TC = a(M_t - M^*{}_t)^2 + b(M_t - M_{t-1})^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Where:
TC = present discounted value of the total cost of adjusting balances

M = actual money balances
M^*= desired money balances.

The buffer stock models (Carr and Darby, 1981; Cuthbertson, 1985; Cuthbertson and Taylor, 1987; and others) posit an intertemporal cost function rather than a one-period one and minimize the present expected value of this cost over the present and future periods. This implies taking account of both types of costs over the present as well as the future periods. Hence, for the first element of cost, the expected cost of future deviations of actual from desired balances, in addition to the cost of a current deviation, is taken into account.

This modification allows the agent to take account of the future levels of desired balances in determining the present amounts held. For the second element of cost, the justification for the intertemporal extension is as follows: just as last period's money holdings affect the cost of adjusting this period's money balances, so would this period's balances affect the cost of adjusting next period's balances, and so on, and these future costs attached to current money balances need to be taken into consideration in the current period. The resulting cost function is intertemporal and forward looking.

The intertemporal extension of (29) for i = 0,1, . . .,T is:

$$TC = \sum_i D^i[a(M_{t+i} - M^*{}_{t+i})^2] + b(M_{t+i} - M_{t+i-1})^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Where:
D = gross discount rate (1/(1+*r*)).
a = the cost of actual balances being different from desired balances
b = the cost of adjusting balances between periods. b can be the brokerage cost of selling bonds but, this can be more than just a monetary cost.

The economic agent is assumed to minimize TC with respect to M_{t+i}, i = 0,1, . . . ,T. Its Euler condition for the last period T is:

$$\partial TC/\partial M_{t+T} = 2a(M_{t+T} - M^*{}_{t+T}) + 2b(M_{t+T} - M_{t+T-1}) = 0$$

so that:

$$M_{t+T} = \frac{a}{a+b}M^*{}_{t+T} + \frac{b}{a+b}M_{t+T-1} = A_1M^*{}_{t+T} + B_1M_{t+T-1} \quad \ldots\ldots\ldots\ldots (31)$$

Where A_1/(a+b) and A_1+B_1 =1. For i <T, the first-order cost-minimizing conditions are:

$$\frac{\partial C}{\partial M_{t+i}} = 2a(M_{t+i} - M^*{}_{t+i}) + 2b(M_{t+i} - M_{t+i-1}) - 2b(M_{t+i+1} - M_{t+i}) = 0$$

$$M_{t+i} = \frac{a}{a+2b}M^*{}_{t+i} + \frac{b}{a+2b}M_{t+i-1} + \frac{b}{a+2b}M_{t+i+1} = A_2M^*{}_{t+i} + B_2M_{t+i-1} + B_2M_{t+i+1} \quad (32)$$

where A_2+2B_2 =1. In (32), both the future and past values of actual balances M, as well as the future values of M^*, affect the demand for money in each period. Eqn (32) implies that:

$$M_t = q_1 M_{t+i-1} + (a/b) q_1 \sum_i q^i{}_t \, M^*{}_{t+i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(33)}$$

where $q_1 + q_2 = (a/b)+2$ and $q_1 q_2 = 1$. The demand function for the
desired money balances M^* is assume it to be:

$$\frac{M^*{}_{t+i}}{p_{t+i}} = b_y y_{t+i} + b_R R_{t+i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(34)}$$

Further, in the context of uncertainty and using the expectations operator E_{t-1} for expectations held in t−1, let:

$$M_t = E_{t-1} M_t + M^u{}_t + \mu_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(35)}$$

where $M^u{}_t$ has been introduced to take account of errors in the expected value of $M^*{}_{t+i}$ due to unexpected changes in its determinants in (34). From (33) to (35),

$$M_t = q_1 M_{t-1} + (a/b) q_1 \sum_i q^i{}_i \{b_y y_{t+i} + b_R R_{t+i}\} p_t M^u{}_t + \mu_t \quad \ldots\ldots\ldots\ldots\ldots \text{(36)}$$

In conclusion, from eqn 36, the actual demand for money depends upon the future and current values of income and interest rates, which shows the model to be a forward-looking one. It also depends upon the lagged value of money balances, thus incorporating a backward-looking element. The model is, therefore, both forward and backward looking.

THE KANNIAINEN AND TARKKA SMOOTHING MODEL (1986)

An alternative version of the intertemporal adjustment cost function (30) used by Kanniainen and Tarkka (1986) is:

$$TC = E_t \sum_i \left[D^i \{ a(M_{t+i} - M^*{}_{t+i})^2 + b(z_{t+i})^2 \} \right] \quad \text{i= 0, 1, 2, ……} \qquad \text{(37)}$$

Where:
z_t = the "induced" changes in money balances brought about by the agent's own actions
b = the brokerage cost of converting bonds to money
M = nominal balances
M^* = the steady-state desired balances
D = discount factor

The rationale for this specification of the cost function is that while the induced changes in money holdings impose a brokerage cost, the autonomous changes do not since they result from the actions of others. The adjustment in nominal balances in t from those in t−1 occurs due to autonomous and induced changes in t, so that:

$$M_t - M_{t-1} = z_t + x_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(38)}$$

Where:
z_t = induced changes in money holdings
x_t = autonomous changes

Substitute (38) in (37) and, to minimize total cost, set the first-order partial derivatives of the resulting equation with respect to mt+i, i = 0,1,2, . . ., equal to zero. This process yields the Euler equations as:

$$E_tM_{t+i+1} - \beta E_tM_{t+i} + (1+R)E_tM_{t+i-1} = -\alpha E_tM^*{}_{t+i} + E_tx_{t+i+1} - (1-R)E_tx_{t+i} \quad (39)$$

Where:
α = a/Db
β = (1/D){(a/b)+D+1}
i = 0,1,2, ...

Note that (39) represents a large number of equations and shows the extensive information requirements of such models. To determine money demand in period t, the agent must have expectations on the autonomous changes in money holdings in (t+1) and its optimal money balances, with the latter requiring this information for period (t +2), and so on. With new information becoming available each period, the model will require continual recalculation.

Equation (39) is a stochastic second-order difference equation in Et Mt+i. Its roots are:

$$\lambda_1, \lambda_2 = 1/2\,[\beta \mp \{\beta^2 - 4(1+R\}]^{1/2}$$

With λ_1 >0 being the stable root and λ_2 <0 being the unstable one. The latter was ignored by K–T in order to exclude cyclical adjustment. Using the positive root λ_1, the Euler condition becomes:

$$E_tM_{t+1} = \lambda_1 M_t + [\lambda_1\,\alpha/(1+R-\lambda_1)]M^*{}_t - \textstyle\sum_i[\lambda_1/(1+R)]^i\,[E_tx_{t+i-1} - (1+R)E_tx_{t+i}] \quad \ldots(40)$$

In (40), the impact of the autonomous adjustment xt on money demand is given by λ_1, the stable root of (39). This impact is the same whether it was anticipated or not. The impact of future autonomous shocks, on which expectations have to be formed, depends upon the rate of time preference. If this rate is high, these expectations will have to be formed for only some periods ahead. Further, changes in these expectations will shift the money demand function.

Substitute (40) into (39) and solve for M_t , noting that $E_tM^*{}_t = M^*{}_t$. This yields:

$$M_t = \lambda_1 M_{t-1} + \rho M^*{}_t + \lambda_1 x_t + z_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (41)$$

Where:

$$\rho = [\lambda_1\,(a/b)(1+R)/(1+R-\lambda_1)]$$

The weighted sum of the future shocks to net receipts and payments, $z^*{}_t$, is given by:

$$z^*{}_t = -(1-\lambda_1)\textstyle\sum_i\{\lambda_1/(1+R)\}^i\,E_tx_{t+j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

The model can be transformed into real terms by dividing (40) by the current price level p_t . The resulting equation, based on (40), is:

$$lnm_t = \alpha_0 + (1-\lambda_1)lnm^*{}_t + \lambda_1 lnm_{t-1} + \gamma ln\, p_t/p_{t-1} + \lambda_1\, x_t/M_{t-1} + z_t/M_{t-1} \ldots\ldots \quad (43)$$

Where:
$m_t = M_t/P_t$ Real money demand

$m^*{}_t = M^*{}_t/P_t$ Desired money demand

K–T specified the desired demand $m^*{}_t$ as a log-linear function of y_t and R_t , such that:

$$m^*{}_t = \gamma y^{\theta}{}_t + R^{\eta}{}_t \ldots\ldots (44)$$

Where θ and η are parameters. The critical autonomous net payments variable x_t was defined as:

$$x_t = \Delta L_t + \Delta L^g{}_t + B_t \ldots\ldots (45)$$

Where:
L = domestic credit expansion
L^g = government net borrowing from abroad
B = surplus in the balance of payments on current account

On the future values of x_t , the following extrapolative model was assumed:

$$E_i x_{t+i} = x_t(1+\theta)^i$$

where θ can be positive or negative or zero. Assuming z_{t+i} to be proportional to x_{t+i} such that $Z_{t+i} = -\xi X_{t+i}$, K–T use (43) to specify the estimating equation as:

$$lnm_t = \alpha_0 + (1-\lambda_1)lnm^*{}_t + \lambda_1 lnm_{t-1} + \gamma ln\, p_t/p_{t-1} + (\lambda_1 - \xi)\, x_t/M_{t-1} + \mu_t \ldots\ldots (46)$$

Where:
$m_t = M_t/P_t$ Real money demand

$m^*{}_t = M^*{}_t/P_t$ Desired money demand

p_t = current price level

p_{t-1} = Lagged price level

x_t = autonomous net payments

M_{t-1} = Lagged nominal money demand
μ = random noise.

Note that the current autonomous injections of money increase current money holdings through the variable x_t.